Centers of Mass

Section 8.3

1. The masses m_i are located at the points P_i. Find the moments and the center of mass of the system.

$$m_1 = 4, m_2 = 2, m_3 = 4, \qquad P_1 = (2, -3), P_2 = (-3, 1), P_3 = (3, 5)$$

Suppose that $f(x)$ and $g(x)$ enclose a region of area A for $a < x < b$, with $f(x) \geq g(x)$. The center of mass (centroid) of this region is $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{1}{A} \int_a^b x \left[f(x) - g(x) \right] dx \quad \text{and} \quad \bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} \left[(f(x))^2 - (g(x))^2 \right] dx.$$

2. (a) Sketch the region bounded by $x = 0$, $y = 0$, and $y = \sqrt{16 - x^2}$.

   (b) Compute the centroid of this region using the formulas above.

   (c) Why does it make sense that $\bar{x} = \bar{y}$ for the centroid of this region?

3. (a) Sketch the region bounded by $y = \sin(x)$ and $y = 0$ for $0 \leq x \leq \pi$. Visually estimate the centroid of this region.

   (b) Compute the centroid of this region using the formulas above. *Hint:* $\sin^2(x) = \frac{1-\cos(2x)}{2}$

4. Find the centroid of the region bounded by the graphs of $y = x^2$ and $y = x + 2$.

> **Theorem of Pappus:** Let $\mathcal{R}$ be a region that lies entirely on one side of a line ℓ in the plane. If $\mathcal{R}$ is rotated about the line ℓ, then the volume of the resulting solid is the product of the area A of $\mathcal{R}$ and the distance d traveled by the centroid of $\mathcal{R}$.

5. Use the Theorem of Pappus to find the volume of the solid obtained by rotating the triangle with vertices $(2, 1)$, $(4, 1)$, and $(4, 3)$ about the x-axis.