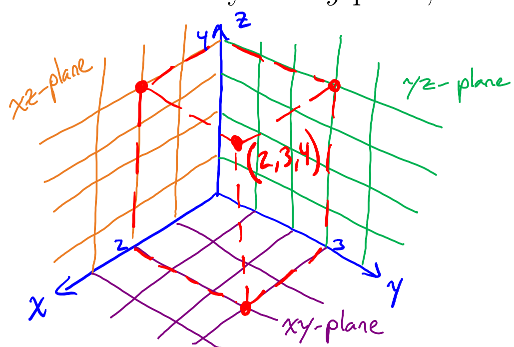
3-D Coordinates and Vectors

Sections 12.1 and 12.2

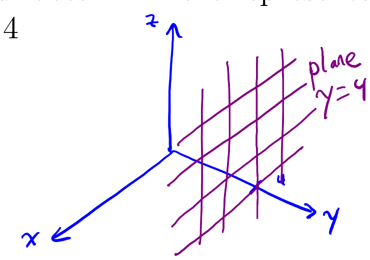
1. Sketch the 3-D coordinate axes. Identify the xy-plane, the xz-plane, and the yz-plane.



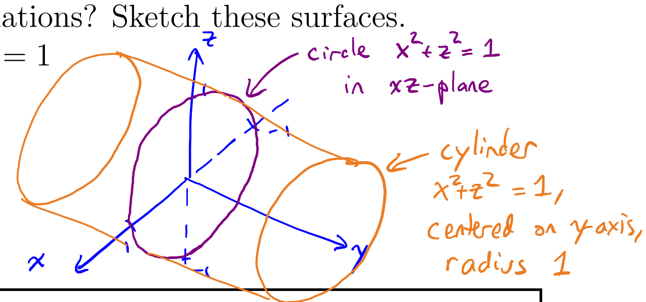
2. Plot the point $(2, 3, 4)$ in your coordinate system above. Also plot the projections of this point onto the xy- and yz-planes.

3. What surfaces in $\mathbb{R}^3$ are represented by the following equations? Sketch these surfaces.

(a) $y = 4$



(b) $x^2 + z^2 = 1$



Distance Formula in 3-D: The distance $|P_1P_2|$ between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)}$$

Consider the points $P = (1, -2, 1)$, $Q = (5, 1, 1)$, and $R = (1, 1, 1)$ in $\mathbb{R}^3$.

4. Find the distance between the points P and Q.

$$|PQ| = \sqrt{(5-1)^2 + (1--2)^2 + (1-1)^2} = \sqrt{4^2+3^2+0^2} = 5$$

5. Find an equation of the sphere with radius 3 centered at P.

Point (x,y,z) is on the sphere if the distance from (x,y,z) to P is 3.

Thus, $\sqrt{(x-1)^2 + (y--2)^2 + (z-1)^2} = 3$, or $(x-1)^2 + (y+2)^2 + (z-1)^2 = 9$.

6. Does R lie within the sphere of radius 2 centered at P?

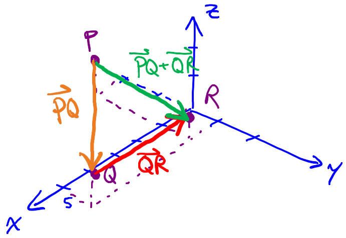
Distance: $|PR| = \sqrt{(1-1)^2 + (1--2)^2 + (1-1)^2} = 3$,

So point R is outside the sphere of radius 2 centered at P

7. Sketch the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. Write each of these vectors in component form. What is the length of each of these vectors?

$\overrightarrow{PQ} = \langle 4, 3, 0 \rangle$

$\overrightarrow{QR} = \langle -4, 0, 0 \rangle$



lengths

$|\overrightarrow{PQ}| = \sqrt{4^2+3^2+0^2} = 5$

$|\overrightarrow{QR}| = \sqrt{(-4)^2+0^2+0^2} = 4$

8. Sketch the vector sum $\overrightarrow{PQ} + \overrightarrow{QR}$, and write this sum in component form.

See sketch above

$\overrightarrow{PQ} + \overrightarrow{QR} = \langle 4, 3, 0 \rangle + \langle -4, 0, 0 \rangle = \langle 0, 3, 0 \rangle$

9. Given vectors $\mathbf{v} = \langle 0, 4, 2 \rangle$ and $\mathbf{u} = \langle 4, 4, 2 \rangle$, find the following:

(a) $\mathbf{u} + \mathbf{v} = \langle 4, 4, 2 \rangle + \langle 0, 4, 2 \rangle = \langle 4, 8, 4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 4, 4, 2 \rangle - \langle 0, 4, 2 \rangle = \langle 4, 0, 0 \rangle$

(c) $2\mathbf{v} + 3\mathbf{u} = 2\langle 0, 4, 2 \rangle + 3\langle 4, 4, 2 \rangle = \langle 0, 8, 4 \rangle + \langle 12, 12, 6 \rangle = \langle 12, 20, 10 \rangle$

(d) $|\mathbf{v} - \mathbf{u}| = \left|\langle 0, 4, 2 \rangle - \langle 4, 4, 2 \rangle\right| = \left|\langle -4, 0, 0 \rangle\right| = \sqrt{(-4)^2+0^2+0^2} = 4$

The following vectors are called the **standard basis vectors** for 3 dimensions:

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$

Any 3-dimensional vector can be expressed in terms of **i**, **j**, and **k**.

Wednesday, April 12

10. Express $\mathbf{v} = \langle 3, -1, 2 \rangle$ in terms of **i**, **j**, and **k**

$\vec{v} = 3\vec{i} - \vec{j} + 2\vec{k}$

11. A **unit vector** is a vector with length 1. Find a unit vector in the direction of $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

$\vec{v} = \langle 1, 1, -2 \rangle \qquad |\vec{v}| = \sqrt{6}$

A unit vector in the direction of $\vec{v}$ is $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{\sqrt{6}} = \left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \right\rangle$