5. Find the angle between the vectors $\langle 1, 0, 1\rangle$ and $\mathbf{i}$.

$$\cos\theta = \frac{\langle 1,0,1\rangle \cdot \langle 1,0,0\rangle}{\sqrt{2}\cdot 1} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

6. Two vectors are **orthogonal** (or perpendicular) if the angle between them is $\frac{\pi}{2}$. If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, then what is $\mathbf{u} \cdot \mathbf{v}$?

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\left(\frac{\pi}{2}\right) = |\vec{u}||\vec{v}|\,0 = 0$$

7. If $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors, how does $\mathbf{u} \cdot \mathbf{v}$ relate to $|\mathbf{u}|$ and $|\mathbf{v}|$?

parallel: $\theta = 0$, so $\cos\theta = 1$

then: $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|$

"component of $\vec{u}$ in the direction of $\vec{v}$"

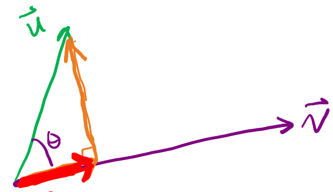
Scalar projection of $\mathbf{u}$ onto $\mathbf{v}$: $\quad \text{comp}_{\mathbf{v}}\mathbf{u} = \dfrac{\vec{v}\cdot\vec{u}}{|\vec{v}|}$

length of the red arrow

Vector projection of $\mathbf{u}$ onto $\mathbf{v}$: $\quad \text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{\vec{v}\cdot\vec{u}}{|\vec{v}|}\cdot\dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\vec{v}\cdot\vec{u}}{|\vec{v}|^2}\vec{v}$

(red vector)

projection of $\vec{u}$ onto $\vec{v}$




projection of $\vec{u}$ onto $\vec{v}$

Started Here on Wednesday:

8. Find the scalar and vector projections of $\mathbf{u} = \langle 2, 3\rangle$ onto $\mathbf{v} = \langle 1, 4\rangle$.

$$\text{comp}_{\vec{v}}\vec{u} = \frac{\langle 2,3\rangle\cdot\langle 1,4\rangle}{|\langle 1,4\rangle|} = \frac{14}{\sqrt{17}}$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{14}{\sqrt{17}}\cdot\frac{\vec{v}}{|\vec{v}|} = \frac{14}{\sqrt{17}}\cdot\frac{\langle 1,4\rangle}{\sqrt{17}} = \frac{14}{17}\langle 1,4\rangle = \left\langle \frac{14}{17}, \frac{56}{17}\right\rangle$$

9. Find the scalar and vector projections of $\mathbf{u} = 2\mathbf{j} + \mathbf{k}$ onto $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

$$\text{comp}_{\vec{v}}\vec{u} = \frac{\langle 0,2,1\rangle\cdot\langle 2,-1,4\rangle}{|\langle 2,-1,4\rangle|} = \frac{0-2+4}{\sqrt{4+1+16}} = \frac{2}{\sqrt{21}}$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{2}{\sqrt{21}}\,\frac{\langle 2,-1,4\rangle}{\sqrt{21}} = \frac{2}{21}\langle 2,-1,4\rangle = \left\langle \frac{4}{21}, \frac{-2}{21}, \frac{8}{21}\right\rangle$$

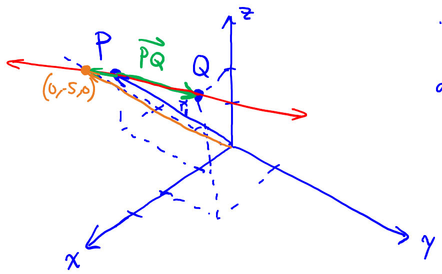
unit vector in direction of $\vec{v}$

10. Show that the vector $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$ is orthogonal to $\mathbf{v}$.

# Lines and Planes in Space

Section 12.5

1. Find the vector, parametric, and symmetric equations of the line in $\mathbb{R}^3$ through the points $P = (1, -2, 1)$ and $Q = (2, 1, 2)$.




initial point: $(1, -2, 1)$

direction vector: $\vec{PQ} = \langle 1, 3, 1 \rangle$

LINE:
- Vector eq: $\vec{r} = \langle 1, -2, 1 \rangle + t\langle 1, 3, 1 \rangle$
- parametric eq: $x = 1+t, \quad y = -2+3t, \quad z = 1+t$
- symmetric eq: $x - 1 = \frac{y+2}{3} = z - 1$

2. At what point does the line in #1 intersect the $xy$-plane?
*Hint*: In the parametric equations, set $z = 0$ and solve for $t$.

$(x, y, 0)$ is a point in the $xy$-plane

$\langle x, y, 0 \rangle = P + t\vec{v}$, or $0 = 1 + t$ is the z-equation (of the parametric equations)

Thus, $t = -1$ gives a point $(x, y, 0)$.

$t = -1$: $\langle x, y, z \rangle = \langle 1, -2, 1 \rangle + (-1)\langle 1, 3, 1 \rangle$
$= \langle 0, -5, 0 \rangle$

3. Find the point where the line in #1 intersects the line given by the parametric equations:

$$x = 4 + s, \ y = 3 - s, \ z = 5 + 2s.$$

*Hint*: Using the parametric equations for the two lines, equate the corresponding expressions for $x$. Then do the same for $y$, and for $z$. Does this give you a system of equations that you can solve for $s$ and $t$?

For some $t$, $\langle 1+t, -2+3t, 1+t \rangle = \langle 4+s, 3-s, 5+2s \rangle$ for some $s$

to be continued...

4. We say that two lines in $\mathbb{R}^3$ are *parallel* if they have the same direction. If they if they are not parallel and they do not intersect, we say the lines are *skew*. Give an example of two skew lines in $\mathbb{R}^3$.
*Hint*: Start with a sketch.