Math 230 B — 6 Oct. 2017

LAST TIME: $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9 = 0$

Solutions: $y_1(t) = e^{-3t}$, $y_2(t) = te^{-3t}$

$$\frac{dy_2}{dt} = e^{-3t} + t(-3e^{-3t}) = (1-3t)e^{-3t}$$

$$\frac{d^2y_2}{dt^2} = (-3)e^{-3t} + (1-3t)(-3e^{-3t}) = (-6+9t)e^{-3t}$$

plug in: $\frac{d^2y_2}{dt^2} + 6\frac{dy_2}{dt} + 9y_2$

$$= \left[(-6+9t)e^{-3t}\right] + 6\left[(1-3t)e^{-3t}\right] + 9te^{-3t}$$

$$= (\cancel{-6} + \cancel{9t} + \cancel{6} \cancel{-18t} + \cancel{9t})e^{-3t} = 0$$

General Solution: $\boxed{y(t) = Ae^{-3t} + Bte^{-3t}}$

SOLVING CERTAIN SYSTEMS

1. $\frac{dx}{dt} = 3x$ $\longrightarrow$ has solution $x(t) = k_1e^{3t}$

$\frac{dy}{dt} = y+2$ $\longrightarrow$ $\int \frac{dy}{y+2} = \int dt$

$\ln|y+2| = t + c$

$e^{\ln|y+2|} = e^{t+c}$

$y+2 = k_2e^t$

$y(t) = k_2e^t - 2$

COMPLETELY DECOUPLED SYSTEM

$$\boxed{\begin{array}{l}\text{Solution to the system:}\\ x(t) = k_1e^{3t}\\ y(t) = k_2e^t - 2\end{array}}$$

2. $\frac{dx}{dt} = 3x - 2y$

$\frac{dy}{dt} = 4y$ $\longrightarrow$ $y(t) = k_2e^{4t}$

plug this into the other diff. eq:

$$\frac{dx}{dt} = 3x - 2(k_2e^{4t})$$

PARTIALLY DECOUPLED SYSTEM

Assoc. homogeneous eq: $\frac{dx}{dt} = 3x$

has solution $x_h(t) = k_1e^{3t}$

$$\boxed{\begin{array}{l}\text{Solution to the system:}\\ x(t) = k_1 e^{3t} - 2k_2 e^{4t}\\ y(t) = k_2 e^{4t}\end{array}}$$

Particular solution: $x_p = A e^{4t}$

Plug in: $\frac{dx_p}{dt} = 3x_p - 2k_2 e^{4t}$

$$4A\cancel{e^{4t}} = 3A\cancel{e^{4t}} - 2k_2 \cancel{e^{4t}}$$

$$4A = 3A - 2k_2$$

$$A = -2k_2$$

Solution: $x(t) = k_1 e^{3t} - 2k_2 e^{4t}$