Final Exam: Take-Home

Name: _____

Math 230: Differential Equations

Due at the final exam session

Take-Home Instructions:

1. *For this part of the exam, you may use your textbook, your notes, the course web site, and computing technology (e.g. Mathematica).*

2. *Do not consult other sources, people, web sites, etc.*

3. *Answer the following questions in the space provided. Check your work.*

4. *Don't forget the pledge on page 4.*

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1. (12 points) Consider the following system:

$$\frac{dx}{dt} = 4y(x - 2) - 2x$$
$$\frac{dy}{dt} = 3x^2 - 3y$$

(a) Find equations of the nullclines of this system.

(b) Find and classify all equilibrium points of this system.

(c) Sketch a graph that clearly shows the nullclines and all equilibrium points.

(d) Either find a Hamiltonian function for this system, or show that the system is not Hamiltonian.

(e) What is the long-term behavior of the solution with initial condition $(x, y) = (-1, -1)$?

2. (8 points) Consider the initial-value problem

$$\frac{dy}{dt} = 3y - 4t - 1, \qquad y(0) = 1.$$

Use the Runge-Kutta method to approximate $y(3)$ with 10, 20, 40, and 80 steps. Then find the exact solution $y(t)$, and compute the error for each of your approximations. Write your results in the following table.

number of steps	step size	approximation of $y(3)$	error
10			
20			
40			
80			

What is the exact solution $y(t)$ to the initial-value problem?

Please attach computational output that justifies at least one of the approximations that you reported in the table above.

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3. (3 points) Has this class helped you develop as a mathematical thinker and problem solver? Has this course helped you develop a growth mindset toward learning and doing mathematics? How so? Explain.

(There are no wrong answers, but you must explain your answer.)