Math 234

Key Concepts of Discrete Mathematics — Day 1

Discuss the following problems with the people at your table.

universal statement

1. Consider the following statements. Identify which statements (i) express that a certain property is true, (ii) express an if-then condition, or (iii) express the existence of something.

   conditional statement — existential statement

   ☞ Bonus: is each statement true?

   (a) All positive numbers are greater than zero.

   Universal (i) — True

   (b) If an integer n is divisible by 6, then n is even.

   Conditional — True

   (c) Every solution to $x^2 - 7x + 12 = 0$ is an integer.

   Universal — $x = 3$ or 4 — True

   (d) There is a prime number larger than 1 billion.

   True

   Existential

   (e) If n is a nonnegative integer, then $n^2 + n + 41$ is prime.

   False

   Conditional — $n=41$: $41^2+41+41 = 41(41+1+1) = 41(43)$ not prime

   (f) If r and s are distinct rational numbers, then there exists another rational number x between r and s.

   Conditional, Existential — True

2. After each of the following statements there are incomplete statements with missing elements. Fill in the blanks to make each incomplete statment equivalent to the original statement.

   (a) *There is an integer that has remainder* 2 *when divided by* 5 *and remainder* 3 *when divided by* 6.

   ☞ Is this true?

   - There is an integer n such that <u>$n \% 5 = 2$ and $n \% 6 = 3$</u>.

     modulus (remainder)

   - There exists <u>an integer n</u> such that if n is divided by 5 the remainder is 2 and <u>if n has remainder 3 when divided by 6.</u>

   (b) *The cube root of any negative number is negative.*

   - Given any negative real number s, <u>the cube root of s is negative</u>.
   - For any real number s, if <u>$s < 0$</u>, then <u>$\sqrt[3]{s} < 0$</u> ____________.
   - If a real number s <u>is negative</u>, then <u>the cube root of s is negative</u>.

3. Come up with your own examples of each type of statement.

   (a) Universal statement:

   (b) Conditional statement:

   (c) Existential statement:

Sets are important for our study of discrete mathematics. A **set** is a collection of objects, called *elements*. A set may be specified by writing its elements within curly braces, such as $\{1, 2, 3\}$.

A set is completely determined by its elements, not by the order in which they are written or whether some element is written more than once.

We often use the following symbols to denote common sets of numbers:

- **R** or $\mathbb{R}$: the set of all real numbers
- **Z** or $\mathbb{Z}$: the set of all integers
- **Q** or $\mathbb{Q}$: the set of all rational numbers (that is, fractions of integers)

4. Discuss the following questions about sets with the people at your table.

   (a) Let $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$, and $C = \{1, 1, 2, 3, 3, 3\}$. What are the elements of A, B, and C? How are these sets related?

   $A = B = C$

   (b) Which of the following sets is equal to A?

   $\{x \in \mathbf{R} \mid 0 < x < 4\}$ $\qquad$ $\{x \in \mathbf{Z} \mid 0 < x < 4\}$ $\qquad$ $\{x \in \mathbf{Z} \mid 0 \le x \le 4\}$

   set-builder notation

   (c) Is $\{0\} = 0$?

   The set containing zero is distinct from zero.

   (d) How many elements are in the set $\{1, \{1\}\}$?

   2

Set A is a **subset** of set B, written $A \subseteq B$, if every element in A is also in B. Set A is a **proper subset** of B if A is a subset of B and B also contains some element(s) not in A.

$\mathbf{Z}^+ = \{1, 2, 3, \dots\}$

5. Let $A = \mathbf{Z}^+$, $B = \{n \in \mathbf{Z} \mid 0 \leq n \leq 100\}$, and $C = \{100, 200, 300, 400, 500\}$.

   Discuss whether each of the following statements is true or false.

   (a) $B \subseteq A$ False, since zero is in B but not in A

   (b) C is a proper subset of A

   True

   (c) C and B have at least one element in common

   True

   (d) $C \subseteq B$ False

   (e) $C \subseteq C$ True

☞ $\mathbf{Z}^+$ is the set of *positive* integers

Given sets A and B, the **Cartesian product of** A **and** B, denoted $A \times B$ and read "A cross B," is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$. Symbolically:

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

6. Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$. Describe each of the following sets:

   (a) $A \times B$ $\{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$ 6 elements total

   (b) $B \times A$ $\{(x,1), (x,2), (x,3), (y,1), (y,2), (y,3)\}$

   (c) $A \times A$ 9 elements $\{(1,1), (1,2), (1,3), (2,1), \dots, (3,3)\}$

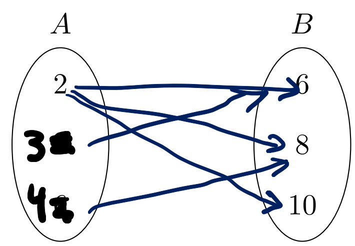
   (d) $\mathbf{Z} \times A$ $\{(n,a) \mid n \in \mathbb{Z} \text{ and } a \in A\}$

7. Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$. Define R to be the set of all $(a, b) \in A \times B$ such that $\frac{b}{a}$ is an integer.

   (a) Write all ordered pairs (a, b) that are in R.

   $R = \{(2,6), (2,8), (2,10), (3,6), (4,8)\}$

   (b) Draw an arrow from $a \in A$ to $b \in B$ if $(a, b) \in R$.



Arrow diagram for a relation of two sets.

👈 You have made an *arrow diagram* for relation R!

> Let A and B be sets. A **relation** R **from** A **to** B is a subset of $A \times B$. If $(a, b) \in R$, we say that x is related to y by R, which we sometimes write as $x \, R \, y$. The set A is called the **domain** of the relation, and B is called the **co-domain** of the relation.
>
> A relation is a **function** if *each* element of the domain is paired with *exactly one* element of the co-domain.

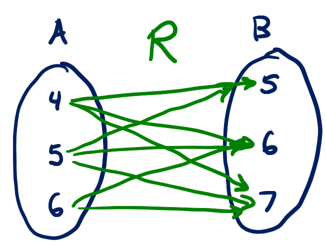
8. Let $A = \{4, 5, 6\}$, $B = \{5, 6, 7\}$. Define relations R, S, and T as follows:

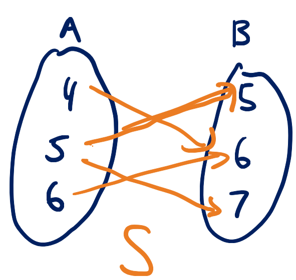
   $(x, y) \in R$ means that $x \leq y$

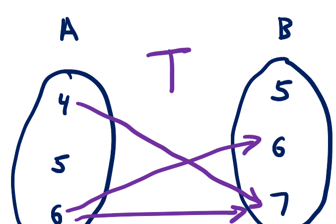
   $(x, y) \in S$ means that $\frac{x-y}{2}$ is an integer

   $T = \{(4, 7), (6, 5), (6, 7)\}$

   (a) Draw arrow diagrams for R, S, and T.







   (b) Determine whether any of the relations R, S and T are functions.

   None of R, S, and T are functions.

   (c) Define your own function with domain A and co-domain B.