

Random Walk Project

Math 242

due Friday, April 19

Consider the following two-dimensional random walk that is not confined to integer-valued coordinates: the walk starts at the origin, and each step is a randomly chosen unit vector. That is, to determine a step of the walk, first choose an angle θ (uniformly) from the interval $[0, 2\pi)$. The step of the walk will then be $(\cos(\theta), \sin(\theta))$.

Your Task

Investigate the following questions:

1. What is the average squared distance from the origin after n steps?
2. When does the walk first return near the origin? Since the walk is not on a grid, it's very unlikely that it will return *exactly* to $(0, 0)$. Instead, when does it return to a small circle around the origin? For example, what is the average number of steps until the walk returns to a circle of radius $\frac{1}{2}$ around the origin?
3. How does the walk behave if it is constrained to the region $-10 \leq y \leq 10$? Modify your code to keep the y -coordinate of the walk between -10 and 10 . Then reconsider questions 1 and 2 for this modified random walk.

Your Report

Turn in *either* a Mathematica notebook or a Python Colab notebook saved as a PDF file. To save your Colab notebook as a PDF file, go to File \rightarrow Print, and choose destination "Save as PDF". Make sure that you clearly answer the questions above, and include computations to support your answers. As usual, submit code that runs and explain what your code does. Your goal should be to communicate your work to another person (e.g., another student at your level who is not in this course).

Grading Rubric

Your notebook will be graded on a scale of 0 to 16 points. The following rubric gives characteristics of notebooks that will merit sample point totals. (Interpolate the following for point totals that are not divisible by 4.)

- 16 points.** Questions and goals are clearly stated, including relevant definitions or parameters. Computations are complete; code runs and is clearly explained. Conclusions are clearly stated and backed up by sufficient computational evidence. Limitations of the methodology, extensions for future work, and conjectures are discussed. Notebook is well-formatted and easy to read.
- 12 points.** Questions and goals are stated well, though relevant definitions or parameters may be missing. Computations are mostly complete; code runs, but explanation is weak. Conclusions are unclear or not well justified. Insufficient discussion of limitations, extensions, and conjectures.

- 8 points.** Statement of questions or goals is unclear. Computations are incomplete; explanation is ambiguous. Code may produce errors when run. Conclusions are possibly correct, but not justified. Little or no discussion of limitations, extensions, or conjectures. Notebook is difficult to read.
- 4 points.** Serious misunderstanding of the questions or goals. Computation is inadequate for the task at hand. Work is not clearly explained. No discussion of limitations, extensions, or conjectures. Notebook is difficult to read.
- 0 points.** Notebook is not turned in.