Last time, we looked at $\frac{F_{n+1}}{F_n} \to \frac{1+\sqrt{5}}{2}$ golden ratio

why? Assume that $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$ exists. ← ?

Consider the recursive definition: $F_{n+1} = F_n + F_{n-1}$

Divide by F_n: $\frac{F_{n+1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$

Take limit: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 1 + \lim_{n\to\infty} \frac{F_{n-1}}{F_n}$

Let $x = \lim_{n\to\infty} \frac{F_{n+1}}{F_n}$. Then: $x = 1 + \frac{1}{x}$

$$x^2 = x + 1$$

We get a quadratic: $x^2 - x - 1 = 0$

Then: $x = \frac{1 \pm \sqrt{1-4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$

The positive case is golden ratio: $\frac{1+\sqrt{5}}{2} = \phi$

Investigate $F_n^2 - F_{n+1} F_{n-1}$

$\cancel{F_1}^2 - \cancel{F_2}\,\cancel{F_0}$

$F_2^2 - F_3 F_1 = -1$

$F_3^2 - F_4 F_2 = 1$

$F_4^2 - F_5 F_3 = -1$

$\vdots$

$F_{\cancel{10}\,n}^2 - F_{\cancel{11}\,n+1} F_{\cancel{9}\,n-1}$

fibSquared − fibA * fibB

Investigate: $F_n^2 - F_{n+r} F_{n-r}$

Pick a value of r. Then look for a pattern as n varies.

Catalan's Identity: $F_n^2 - F_{n-r} F_{n+r} = (-1)^{n-r} F_r^2$

For Monday, verify this for several values of r and lots of values of n.