

Last time, we looked at $\frac{F_{n+1}}{F_n} \rightarrow \frac{1+\sqrt{5}}{2}$ golden ratio

why? Assume that $\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$ exists. $\leftarrow ?$

Consider the recursive definition: $F_{n+1} = F_n + F_{n-1}$

Divide by F_n : $\frac{F_{n+1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$

Take limit: $\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = 1 + \lim_{n \rightarrow \infty} \frac{F_{n-1}}{F_n}$

Let $x = \lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$. Then: $x = 1 + \frac{1}{x}$

$$x^2 = x + 1$$

We get a quadratic: $x^2 - x - 1 = 0$

Then: $x = \frac{1 \pm \sqrt{1 - 4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$

The positive case is golden ratio: $\frac{1+\sqrt{5}}{2} = \phi$

Investigate $F_n^2 - F_{n+1}F_{n-1}$

The diagram shows the sequence of values for the expression $F_n^2 - F_{n+1}F_{n-1}$. The terms are: $F_1^2 - F_2F_0$, $F_2^2 - F_3F_1 = -1$, $F_3^2 - F_4F_2 = 1$, $F_4^2 - F_5F_3 = -1$, and so on. The terms are grouped into three vertical columns: a yellow column containing F_1^2 , F_3^2 , F_5^2 , and F_{2n-1}^2 ; a green column containing F_2^2 , F_4^2 , F_6^2 , and F_{2n}^2 ; and a pink column containing F_0 , F_1 , F_2 , F_3 , and F_{n-1} . Arrows at the bottom point to these columns with labels: 'fibSquared' for the yellow column, 'fibA * fibB' for the green column, and 'fibB' for the pink column.

Investigate: $F_n^2 - F_{n+r} F_{n-r}$

Pick a value of r . Then look for a pattern as n varies.

Catalan's Identity: $F_n^2 - F_{n-r} F_{n+r} = (-1)^{n-r} F_r^2$

For Monday, verify this for several values of r
and lots of values of n .