

MEAN - MEDIAN MAP

Start with x_1, x_2, \dots, x_n .

Compute: x_{n+1} such that

$$\frac{x_1 + \dots + x_n + x_{n+1}}{n+1} = \text{median}(x_1, \dots, x_n)$$

Repeat.

THEOREM: The sequence of medians is monotone.

(always increasing or always decreasing)

Start with $S_n = \{x_1, \dots, x_n\}$.

$$\text{Compute: } x_1 + \dots + x_n + x_{n+1} = (n+1) \text{median}(S_n) \quad (1)$$

$$x_1 + \dots + x_n + x_{n+1} + x_{n+2} = (n+2) \text{median}(S_{n+1}) \quad (2)$$

Subtract (1) from (2):

$$x_{n+2} = (n+2) \text{median}(S_{n+1}) - (n+1) \text{median}(S_n)$$

$$x_{n+2} = (n+1) \left(\text{median}(S_{n+1}) - \text{median}(S_n) \right) + \text{median}(S_{n+1})$$

If $\text{median}(S_{n+1}) \geq \text{median}(S_n)$, then $\text{median}(S_{n+1}) - \text{median}(S_n)$ is nonnegative, so

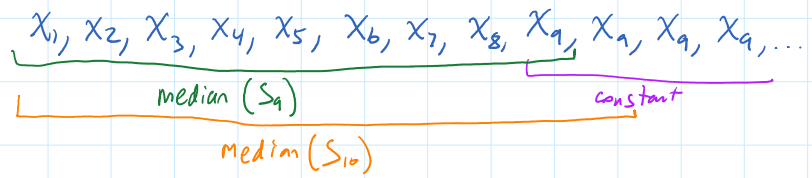
$x_{n+2} \geq \text{median}(S_{n+1})$. Thus, by definition of median,

$\text{median}(S_{n+2}) \geq \text{median}(S_{n+1})$. We have a non-decreasing sequence of medians.


If $\text{median}(S_{n+1}) \leq \text{median}(S_n)$, then similarly, we get a non-increasing sequence of medians.

OBSERVATION

Sequence of values:



If two consecutive medians are equal, then the sequence of values is constant.

 proof follows from the proof above