Pell Project

Math 242

due Monday, February 24

Continue the exploration of Pell and Lucas numbers as described in the paper "On Polynomial Identities for Recursive Sequences" (`https://arxiv.org/pdf/1508.04953.pdf`). Specifically:

1. Generate, by solving systems of equations, several other polynomial identities similar to those described in Proposition 1. For example, can you directly conjecture and verify the identity for $n = 9$?
2. Provide plentiful computational evidence to confirm two of the following results from the paper:
   - Proposition 2
   - Theorem 2 — Note that $\binom{2m+1}{j}$ is a *binomial coefficient.*
   - Theorem 3
   - Proposition 3 — Note that this generalizes Proposition 1 by adding a parameter to the recurrence formula. Here, $A_0 = 0$, $A_1 = 1$, and $A_{n+1} = sA_n + A_{n-1}$ for $n \geq 1$. (The authors sort of explain this Equation (31) in the paper, but that equation contains a typo.)

Note that each of the results involves one or more parameters, such as n. Don't just settle for checking the result for a few values of n. You have substantial computing power available to you, so try to provide a *lot* of computational evidence for each result. For example, can you check the result for 1,000 values of n? How about 1,000,000 values or more?

Your Mathematica notebook should indicate not only what you computed, but also that you understand what you did. A list of calculations with no reasoning will not suffice. Your goal should be to communicate your solution to another person (e.g., another student at your level who is not in this course).

Only submit code that actually runs. If you can't get something complicated to work, try something simpler. It's better to turn in an incomplete assignment that runs instead of a "complete" assignment that doesn't run.

Your notebook will be graded on a scale of 0 to 16 points. The following rubric gives characteristics of notebooks that will merit sample point totals. (Interpolate the following for point totals that are not divisible by 4.)

16 points. Problems and goals are clearly stated, including relevant definitions or parameters. Computations are complete; code runs and is clearly explained. Conclusions are clearly stated and backed up by sufficient computational evidence. Limitations of the methodology, extensions for future work, and conjectures are discussed. Notebook is well-formatted and easy to read.

12 points. Problems and goals are stated well, though relevant definitions or parameters may be missing. Computations are mostly complete; code runs, but explanation is weak. Conclusions are unclear or not well justified. Insufficient discussion of limitations, extensions, and conjectures.

8 points. Statement of problem or goal is unclear. Computations are incomplete; explanation is ambiguous. Code may produce errors when run. Conclusions are possibly correct, but not justified. Little or no discussion of limitations, extensions, or conjectures. Notebook is difficult to read.

4 points. Serious misunderstanding of the problem or goal. Computation is inadequate for the task at hand. Work is not clearly explained. No discussion of limitations, extensions, or conjectures. Notebook is difficult to read.

0 points. Notebook is not turned in.