LITERAL IMPLEMENTATION: Sieve of Eratosthenes

indexes 1 2 3 4 5 6 7 8 9

nums= $\{2, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}, \cancel{9}, \cancel{10}, 11, \cancel{12}, 13, \cancel{14}, \cancel{15}, \cancel{16}, 17\}$

loop: i from 1 to end

loop: j from i+1 to end

if nums[i] divides nums[j], then delete nums[j]

FAST IMPENTATION:

nums = $\{2, 3, \ldots, n\}$

loop: k from 2 to $\sqrt{n}$:

$$\text{nums}[\![\underset{\text{start}}{k^2} ; \underset{\text{stop}}{n} ; \underset{\text{step}}{k}]\!] = 0$$

Observation: When we remove multiples of k, it suffices to start at k^2. All smaller multiples of k have already been considered

SIEVE OF SUNDARAM

1. Start with a positive integer n.
2. Let list1 contain all integers of the form

$$i + j + 2ij,$$

where i and j are integers, $1 \le i \le j$ and $i+j+2ij \le n$.

example: If $i=1$, $j=1$, then $i+j+2ij = 1+1+2(1)(1) = 4$

so 4 is in list1

3. Let list2 contain all integers $\leq n$ that are not in list1.

example: 3 and 5 are in list2, 4 is not

4. For each number in list2, double it and add 1.
This gives a list of all odd primes up to $2n+1$.

Why does it work?

- The resulting numbers are odd integers.
- Excluded numbers are of the form $q = 2\underbrace{(i+j+2ij)}_{\text{list 1}}+1$

exactly the composite odd numbers

$$q = 4ij + 2i + 2j + 1$$

$$q = (2i+1)(2j+1)$$