1: 5, 13, 17, ..

3: 7, 11, 19, ...

---

let p be a prime $> 10{,}000$

take $n = 2 \cdot 3 \cdot 5 \cdots p$

then: $n+2,\ n+3,\ n+4,\ \ldots,\ n+10000$ are all composite

SIEVE OF ERATOSTHENES

LITERAL IMPLENTATION

$$\begin{array}{l}
\text{index:} \quad 1 \quad 2 \quad\quad 3 \quad\quad 4 \quad 5 \quad 6 \quad 7 \\
\text{nums} = 2,\ 3,\ \cancel{4},\ 5,\ \cancel{6},\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ \ldots\ ,\ n
\end{array}$$

loop: i from 1 to end

loop: j from i+1 to end

if nums[i] divides nums[j], then delete nums[j]

FAST ERATOSTHENES

$$\begin{array}{l}
\text{indexes:} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \\
\text{nums} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots, n\}
\end{array}$$

loop: k from 2 to $\sqrt{n}$:

$\text{nums}[[\underset{\text{start}}{k^2};\ \underset{\text{end}}{n};;\ \underset{\text{step size}}{k}]] = 0$ ← set all multiples of k to zero

observation: it suffices to start at k^2, since any smaller multiple of k is already set to zero

SIEVE OF SUNDARAM

1. Start with a positive integer n.

2. list1 = all integers of the form

$$i+j+2ij$$

where i and j are integers, $1 \le i \le j$, and $i+j+2ij \le n$.

example: if $i=j=1$, then $i+j+2ij = 1+1+2 = 4$

so 4 is in list1

3 and 5 are not in list1

3. Let list2 consist of all integers $\le n$ that are not in list1.

e.g.: 3 and 5 are in list2, but 4 is not

4. Take each number in list2, double it and add 1. This gives a list of all odd primes up to $2n+1$.

Why does this work?

The numbers that result from this algorithm are clearly odd integers.

The excluded numbers are of the form

$$
\begin{aligned}
q &= 2(i+j+2ij)+1 \\
&= 4ij+2i+2j+1 \\
&= (2i+1)(2j+1) \leftarrow \text{composite}
\end{aligned}
$$