PRIMES CONJECTURES

- **Conjecture A.** Every even integer greater than 2 is the sum of two primes.

  Goldbach's conjecture   not proved, no counterexample know.

- **Conjecture B.** For every N, the number of nonnegative integers less than N with an *even* number of prime factors is less than the number of nonnegative integers less than N with an *odd* number of prime factors. For this, prime factors are counted *with multiplicity*; e.g., $24 = 2^3 \cdot 3$ has 4 prime factors, while $588 = 2^2 \cdot 3 \cdot 7^2$ has 5 prime factors.

  Polya Conjecture   first counterexample: $N = 906{,}150{,}257$

- **Conjecture C.** For every positive integer n, there exists at least one prime between n^2 and $(n+1)^2$.

  Legenre's Conjecture

  not proved, no counterexample known

- **Conjecture D.** All odd numbers greater than 1 are either prime, or can be expressed as the sum of a prime and twice a square.

  Counterexamples: 5777 and 5993

PRIME COUNTING FUNCTION:

$\pi(x)$ is the number of primes $\leq x$

example: $\pi(5) = 3$

$\pi(10) = 4$

Fast implementation:

$$\text{primelist} = \{\underset{1}{2}, \underset{2}{3}, \underset{3}{5}, \underset{4}{7}, \underset{5}{11}, \underset{6}{13}, \underset{7}{17}, \underset{\dots}{\dots}, n\}$$

$c = 1\ 2\ 3\ 4\ 5$

$$\text{counts} = \{\underset{1}{0}, \underset{2}{1}, \underset{3}{2}, \underset{4}{2}, \underset{5}{3}, \underset{6}{3}, \underset{7}{4}, \underset{8}{4}, \underset{9}{4}, \underset{10}{4}, \underset{11}{5}, \underset{\dots}{\dots}\}$$

$i =$

Density of primes near x:

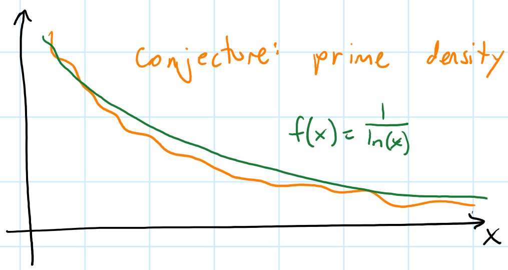
What proportion of numbers near x are prime?

example: $x = 1000$

$$\text{density} \approx \frac{\pi(1020) - \pi(980)}{40}$$

$$\text{density} \approx \frac{\pi(1040) - \pi(1000)}{40}$$

$$\text{density} \approx \frac{\pi(x+d) - \pi(x)}{d} \qquad d > 0$$



conjecture: prime density function looks like this

example:

primeDensity[x_, d_] := Module[{ }...]

returns density at x

(1) densityList = Table[primeDensity[n, 50], {n, 1, 1000}]

ListPlot[densityList]

(2) Plot[densityList[Floor[x], 50], {x, 0, 1000}]