PRIMES CONJECTURES

- **Conjecture A.** Every even integer greater than 2 is the sum of two primes.
  GOLDBACH'S CONJECTURE — no counterexample known
- **Conjecture B.** For every N, the number of nonnegative integers less than N with an *even* number of prime factors is less than the number of nonnegative integers less than N with an *odd* number of prime factors. For this, prime factors are counted *with multiplicity*; e.g., $24 = 2^3 \cdot 3$ has 4 prime factors, while $588 = 2^2 \cdot 3 \cdot 7^2$ has 5 prime factors.
  POLYA'S CONJECTURE — counterexample: 906,150,257
- **Conjecture C.** For every positive integer n, there exists at least one prime between n^2 and $(n+1)^2$. LEGENRE'S CONJECTURE — no counterexample known
- **Conjecture D.** All odd numbers greater than 1 are either prime, or can be expressed as the sum of a prime and twice a square.
  counterexamples: 5777 and 5993

PRIME COUNTING FUNCTION:

$\pi(x)$ = number of primes $\leq x$

examples: $\pi(5) = 3$ — primes: 2, 3, 5

$\pi(12) = 5$ — 2, 3, 5, 7, 11

Fast implementation:

primes list = $\{\underset{1}{2}, \underset{2}{3}, \underset{3}{5}, \underset{4}{7}, \underset{5}{11}, \underset{6}{13}, \underset{7}{17}, \ldots, \quad \}$

$c = 4$

counts = $\{\underset{1}{0}, \underset{2}{1}, \underset{3}{2}, \underset{4}{2}, \underset{5}{3}, \underset{6}{3}, \underset{7}{4}, \underset{8}{4}, \underset{9}{4}, \ldots, \underset{n}{0}\}$

i $\quad i=5$

Density of primes near x:

What fraction of numbers near x are prime?

one idea: $$\frac{\pi(x+d) - \pi(x)}{d}$$ $\leftarrow$ number of primes p, $x < p \le x+d$

or: $$\frac{\pi(x+d) - \pi(x-d)}{2d}$$

Gauss:
prime density $\approx \dfrac{1}{\log(x)}$

primeDensity[x_, d_] := Module[{ },
(* use stored values $\pi(x)$ to estimate $\dfrac{\pi(x+d) - \pi(x)}{d}$ *)

]

densityVals = Table[primeDensity[n, $\underset{\uparrow \atop d}{100}$], {n, 0, 1000}]

Note: Floor[x] rounds x down to an integer