Do simple symmetric random walks return to the origin?

1D: Yes! $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{\pi k}}$ diverges (∞)

expected number of returns to the origin

prob. that the walk is at the origin at step 2k

2D: A 2D r.w. is something like a pair of 1D walks.

Yes! Prob. that a 2D random walk is at the origin at step 2k

$$\left(\frac{1}{\sqrt{\pi k}}\right)^2 = \frac{1}{\pi k}$$

Expected num. of returns to the origin: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\pi k}$ DIVERGES

so 2D rws return to the origin

RECALL: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges iff $p > 1$

3D: Prob. that a 3D r.w. is at the origin at step 2k: $\left(\frac{1}{\sqrt{\pi k}}\right)^3$

Expected number of returns to origin: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(\pi k)^{3/2}}$ converges

Some 3D rw's return to the origin and some don't.

PERCOLATION THEORY

- Start with a grid of squares
- Decide whether each square is open or closed with probability p. $(0 \le p \le 1)$
- Imagine pouring water on the top of the grid.
- **Question:** Is there a path for water to flow (percolate) from the top to the bottom of the grid?

  How does this depend on p?

  How does this depend on n?