Do all simple symmetric random walks return to the origin?

1D: Yes!

expected number of returns to origin: $\sum_{k=1}^{\infty} \frac{1}{\sqrt{\pi k}}$ DIVERGES (∞)

prob. that r.w. is at origin at step 2k

2D: A 2D is like a pair of 1D walks, in the x- and y-directions

Yes!

expected num. returns to origin: $\sum_{k=1}^{\infty} \frac{1}{\pi k}$ DIVERGES (∞)

prob. that 2D r.w. is at origin at step 2k

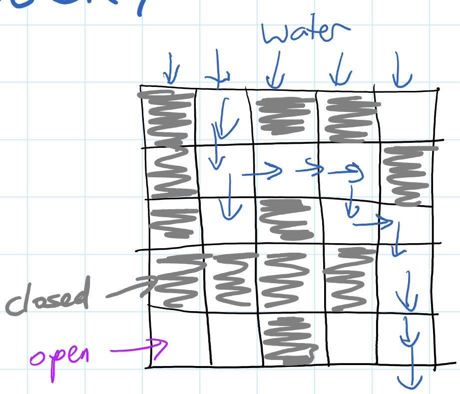
RECALL: $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges iff $p > 1$

3D: expected num. of returns to origin: $\sum_{k=1}^{\infty} \frac{1}{(\pi k)^{3/2}}$ CONVERGES!

No, not all 3D rw's return to the origin.

PERCOLATION THEORY

- Start with a grid of squares
- Let each square be "open" with probability p, and "closed" otherwise.
- Imagine pouring water on the top of the grid.



- Question: Does the water flow to the bottom?

i.e. does a percolation path exist?

How does this depend on the probability p?

How does this depend on the size of the grid?