## 3n Identity: $F_{3n} = a F_n^3 + b F_n^2 + c F_n$

$n=2 \qquad F_6 = a F_2^3 + b F_2^2 + c F_2$

$n=4 \qquad F_{12} = a F_4^3 + b F_4^2 + c F_4$

$n=6 \qquad F_{18} = a F_6^3 + b F_6^2 + c F_6$

↑ nvals

$$\begin{bmatrix} F_6 \\ F_{12} \\ F_{18} \end{bmatrix} = \begin{bmatrix} F_2^3 & F_2^2 & F_2 \\ F_4^3 & F_4^2 & F_4 \\ F_6^3 & F_6^2 & F_6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

↑ Vector ↑ matrix

## 5n Identity: $F_{5n} = a_1 F_n^5 + a_2 F_n^4 + a_3 F_n^3 + a_4 F_n^2 + a_5 F_n$

$n=2$

$n=4$

$n=6$

$n=8$

$n=10$

↑ nvals

$$\begin{bmatrix} F_{10} \\ F_{20} \\ F_{30} \\ F_{40} \\ F_{50} \end{bmatrix} = \begin{bmatrix} F_2^5 & F_2^4 & F_2^3 & F_2^2 & F_2 \\ F_4^5 & & \cdots & & F_4 \\ \vdots & & & & \vdots \\ F_{10}^5 & F_{10}^4 & F_{10}^3 & F_{10}^2 & F_{10} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

↑ vector ↑ matrix ↑ coefficients

## More generally: Fibonacci qn identity, where q is odd

nvals = Range[2, 2q, 2]

vector = Table[f[q*n], {n, nvals}]

matrix = Table[f[n]^i, {n, nvals}, {i, q, 1, -1}]

## Verify Identity: $F_{qn} = a_1 F_n^q + a_2 F_n^{q-1} + a_3 F_n^{q-2} + \dots + a_q F_n$

lhs (for $F_{qn}$); rhs (for $a_1 F_n^q + \dots + a_q F_n$)

Check whether lhs = rhs for $n = 2, 4, 6, \dots, n\text{Max}$.