Math 242 Challenge Problems

Spring 2022

The explorations and exercises below refer to the *Computational Mathematics* text.

1. **Exploration 1.17: π as area.** Implement the algorithm outlined in this exercise (pages 23–25 in the text) to approximate π as the area of a quarter circle. Analyze and discuss the accuracy and efficiency of this method.

2. **Exploration 1.24: Create your own Machin-like formulas**. Use the methods described in Section 1.4 of the text to find your own Machin-like formuas for π. Convert each formula to a power series, implement them in Mathematica, and assess their accuracy for approximating π.

3. **Exploration 1.25: Accuracy of Machin-like formulas.** Investigate the accuracy of Machin-like formuas, such as those in Section 1.4 of the text. Formulate your own conjecture, supported by your computational evidence.

4. **Exercise 1.43: Continued Fractions for π.** Read Section 1.7 in the text. Implement one of the methods for computing convergents of continued fractions. Use your code to complete Exercise 1.43.

5. **Exploration 2.22: Generalize a Fibonacci identity.** Introduce a new index (or more than one) and conjecture a new identity.

6. **Exploration 2.23: Fibonacci identities.** Search for Fibonacci identities based on the three expressions given in the text.

7. **Exploration 2.25: Do algebraic identities lead to Fibonacci identities?** Explore possible Fibonacci identities inspired by the algebraic identities in the text.

8. **Exploration 2.36: Generalized Fibonacci polynomial identity.** Search for a general formula that gives the coefficients of the Fibonacci $(2q+1)n$ identity for odd integers $2q+1$.

9. **Exploration 2.39: Fibonacci identities involving sums.** Explore a class of Fibonacci identities involving $\sum_{a+b=n} F_a F_b$.

10. **Exploration 3.9: Collatz trajectories.** Explore the numbers that arise in Collatz trajectories for certain sets of starting values.

11. **Exploration 3.35: Collatz stopping times.** Explore how the maximum stopping time for Collatz trajectories grows with n.

12. **Exploration 3.36: "Horizontal segments" in Collatz stopping time plot.** Investigate the horizontal line segments that appear in the Collatz stopping time plot. How does the length of these segments increase with n?

13. **Exploration 3.41: Accelerated Collatz trajectories.** Compute accelerated Collatz trajectories for big integers and evaluate the "average" stopping-time estimate on page 99 in the text.

14. **Exploration 3.72: Ergodicity of logistic map trajectories.** How many iterations of the logistic map are required, on average, until the trajectory contains a point in each interval of size $\frac{1}{M}$? How does this depend on M?

15. **A prime polynomial?** The polynomial

$$f(x) = x^2 + x - 1354363$$

produces a lot of prime numbers. For how many integers $n \in [1, 10^4]$ is $|f(n)|$ prime? Is this unusual? Discuss. (For example, does this polynomial produce prime values more frequently than other quadratic polynomials?)

16. **Prime gaps.** A *prime gap* is the difference between successive prime numbers. The first few prime gaps are $1, 2, 2, 4, 2, 4, \ldots$. What is the distribution of prime gaps? Compute the distribution the first n prime gaps for various choices of n and report on your observations. Do you think there is a most common prime gap? Explain.

17. **Fibonacci primes.** Which Fibonacci numbers are prime? Compute the number of Fibonacci primes up to the nth Fibonacci number F_n, for various choices of n. Discuss your observations. Do you think there are infinitely many Fibonacci primes? Explain.