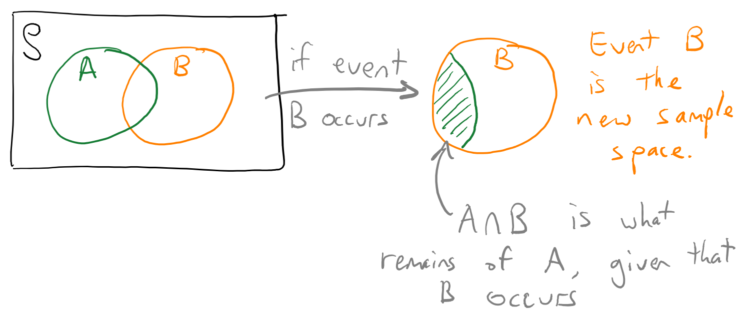
REVIEW: Choosing k items out of n

	ORDER IMPORTANT	ORDER NOT IMPORTANT
WITH REPLACEMENT	n^k	$\binom{k+n-1}{k}$
WITHOUT REPLACEMENT	$\frac{n!}{(n-k)!}$	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

CONDITIONAL PROBABILITY of A GIVEN B:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



1. The probability that Prisca studies for a test is 0.8. The probability that she studies and passes the test is 0.7. If Prisca studies, what is the probability that she passes the test?

Event that Prisca studies: B

Event that Prisca passes: A

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.7}{0.8} = 0.875$$
$$= 87.5\%$$

2. A machine produces parts, 10% of which are defective. An inspector is able to remove 95% of the defective parts. What is the probability that a part is defective *and* removed by the inspector?

$P(\text{removed} \mid \text{defective}) = 0.95$

$P(\text{defective}) = 0.1$

$P(\text{defective} \cap \text{removed}) = P(\text{removed} \mid \text{defective})\, P(\text{defective}) = (0.95)(0.1) = 0.095$

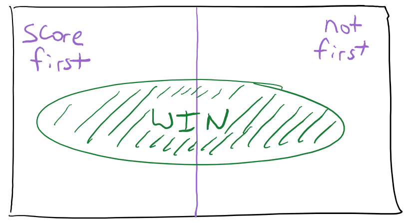
$$\boxed{\text{MULTIPLICATION RULE: } P(A \cap B) = P(A \mid B)\, P(B)}$$

3. A soccer team wins 60% of its games when it scores the first goal, and 30% of its games when the opposing team scores first. If the team scores first in 40% of its games, what percent of its games does it win?

$P(\text{scores first}) = 0.4$

$P(\text{win} \mid \text{scores first}) = 0.6$

$P(\text{win} \mid \text{not first}) = 0.3$



$$P(\text{win}) = P(\text{win} \cap \text{score first}) + P(\text{win} \cap \text{not first})$$

$$= P(\text{win} \mid \text{first})\, P(\text{first}) + P(\text{win} \mid \text{not first})\, P(\text{not first})$$

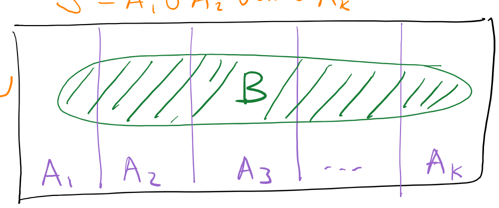
multiplication rule

$$= (0.6)\ (0.4) + P(0.3)\ P(0.6) = 0.42$$

LAW OF TOTAL PROBABILITY:

one of the A_i must occur, $S = A_1 \cup A_2 \cup \dots \cup A_k$

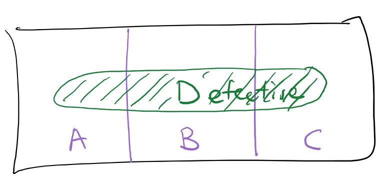
If S is a collection of disjoint, exhaustive events $A_1, A_2, \dots, A_k$, then for any event B,



$$P(B) = \sum_{i=1}^{k} P(B \mid A_i)\, P(A_i).$$

4. A factory uses 3 machines to produce certain items. Machine A produces 50% of the items, 6% of which are defective. Machine B produces 30% of the items, 4% of which are defective. Machine C produces 20% of the items, 3% of which are defective.

(a) What is the probability that a randomly-selected item is defective?



$$P(D) = \underbrace{P(D \mid A)\, P(A)}_{P(D \cap A)} + \underbrace{P(D \mid B)\, P(B)}_{P(D \cap B)} + \underbrace{P(D \mid C)\, P(C)}_{P(D \cap C)}$$

$$= (0.06)(0.5) + (0.04)(0.3) + (0.03)(0.2)$$

$$= \boxed{0.048}$$

(b) If an item is defective, what is the probability that it was produced by Machine A?

$$P(A \mid D) = \frac{P(A \cap D)}{P(D)} \overset{\text{mult. rule}}{=} \frac{P(D \mid A)\, P(A)}{P(D)} = \frac{(0.06)(0.5)}{0.048} = \frac{5}{8} = 0.625$$

"reversed" compared to what we have above

we know this from part (a)

definition of conditional probability

OBSERVE: $P(A|D)P(D) = P(A \cap D) = P(D|A)\,P(A)$

BAYES' THEOREM: $P(A|D) = \dfrac{P(D|A)P(A)}{P(D)}$

5. Suppose that a patient is tested for a disease. Let A be the event that the test is positive, and let D be the event that the patient actually has the disease. Further suppose that:

$P(A \mid D) = 0.99$	(*sensitivity*: probability of a positive test if the patient has the disease)
$P(A' \mid D') = 0.99$	(*specificity*: probability of a negative test if the patient doesn't have the disease)

(a) *Rare disease*: If $P(D) = 0.01$, what is the probability that a patient who tests positive actually has the disease?

to be continued...

(b) *Common disease*: If $P(D) = 0.1$, what is the probability that a patient who tests positive actually has the disease?