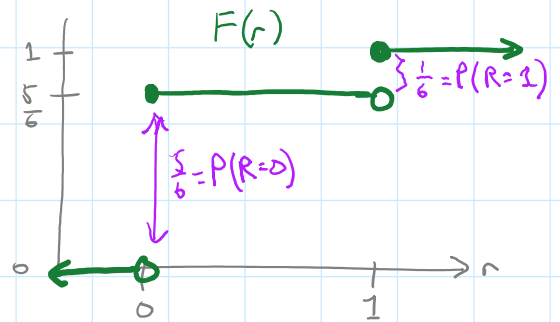
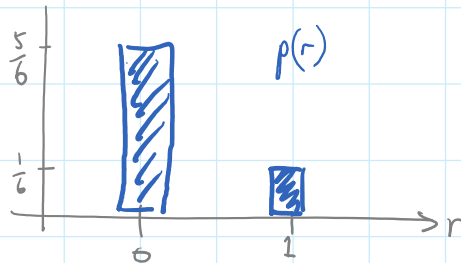


EXPERIMENT: roll two standard, fair dice

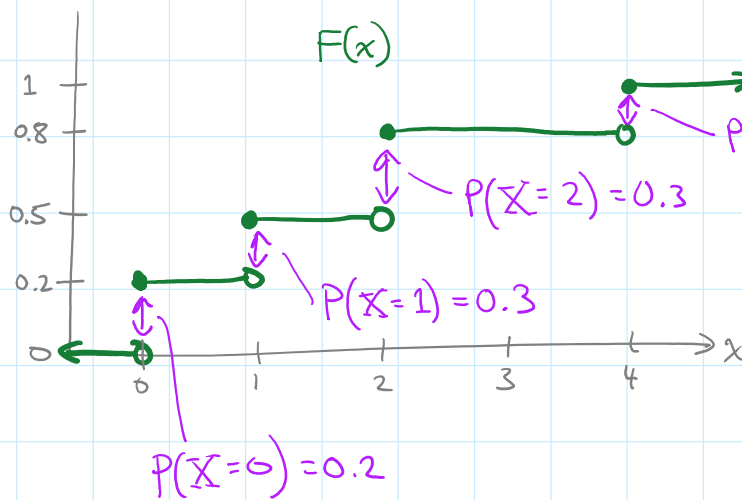
$$R = \begin{cases} 1 & \text{if sum is 7} \\ 0 & \text{otherwise} \end{cases}$$

pmf: $p(1) = P(R=1) = \frac{1}{6}$
 $p(0) = P(R=0) = \frac{5}{6}$

cdf: $F(r) = P(R \leq r)$



2.



random variable X

$$P(X \leq 3.3) = F(3.3) = 0.8$$

Math 262

Sections 2.1 and 2.2

Day 8

1. Let random variable X be the output of `runif(1)` in R (or, if you prefer, the output of `RandomReal[]` in Mathematica). Is X a continuous or discrete random variable?

- (a) Have one person in your group defend the assertion that X is a continuous random variable.
- (b) Have another person in your group defend the assertion that X is a discrete random variable.

- (a) X is equally likely to take any value between 0 and 1, so X is a continuous random variable.
- (b) X is specified by a finite number of decimal digits, so there are only finitely many possible values for X , which makes X a discrete random variable.

X is a discrete approximation of a continuous random variable.

2. The cdf for a random variable X is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \leq x < 1 \\ 0.5 & 1 \leq x < 2 \\ 0.8 & 2 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

- (a) What is $P(X = 2)$?
- (b) What is $P(X = 3)$?
- (c) What is $P(2.5 \leq X)$?
- (d) Sketch the pmf of X .

- (a) $P(X = 2) = F(2) - F(2-) = 0.8 - 0.5 = 0.3$
- (b) $P(X = 3) = F(3) - F(3-) = 0.8 - 0.8 = 0$
- (c) $P(2.5 \leq X) = 1 - F(2.5-) = 1 - 0.8 = 0.2$
- (d) The nonzero values of the pmf are $p(0) = 0.2$, $p(1) = 0.3$, $p(2) = 0.3$, and $p(4) = 0.2$.

3. Each of the following functions might be the pmf for some random variable X . *Discuss with your group:* How can you determine whether a given function is a pmf? Which of these functions is a pmf?

- (a) $p(x) = 2 - 3x$ for $x = 0, 1$
- (b) $p(x) = \frac{x^2}{50}$ for $x = 1, 2, \dots, 5$
- (c) $p(x) = \log_{10}\left(\frac{x+1}{x}\right)$ for $x = 1, 2, \dots, 9$

- (a) The function in (a) is not a pmf because $p(1) = -1$, and probabilities cannot be negative.
- (b) The function in (b) is not a pmf because $\sum_{x=1}^5 \frac{x^2}{50} = \frac{55}{50} \neq 1$, and probabilities must add up to 1.
- (c) The function in (b) is a pmf. The sum of its nonzero values is 1:

$$\sum_{x=1}^9 \log_{10}\left(\frac{x+1}{x}\right) = \log_{10}\left(\frac{2}{1} \cdot \frac{3}{2} \cdots \frac{10}{9}\right) = \log_{10}(10) = 1$$

(This is the pmf for the distribution known as Benford's Law.)

4. Which of the following properties must hold for any cdf $F(x)$? Discuss each property with your group. Either say why it must hold or give a counterexample to show that it might not hold.

(a) $\lim_{b \rightarrow -\infty} F(b) = 0$

(b) $\lim_{b \rightarrow \infty} F(b) = 1$

(c) $F(x)$ is continuous

(d) $F(x)$ is nondecreasing; that is, if $a < b$, then $F(a) \leq F(b)$

(e) $F(b) = 0.5$ for some value b

(a) Yes, because $P(X \leq b) \rightarrow 0$ as $b \rightarrow -\infty$.

(b) Yes, because $P(X \leq b) \rightarrow 1$ as $b \rightarrow \infty$.

(c) No — in the problems above, we have seen many cdfs that are not continuous functions. The cdf for a continuous random variable is continuous, and we'll study those in Chapter 3.

(d) Yes. If $a < b$, then $F(a) = P(X \leq a) \leq P(X \leq b) = F(b)$.

(e) No — consider the cdf in #2(a) above.

★ **BONUS:** Three balls are randomly selected (without replacement) from an urn containing 20 balls numbered 1 through 20. Let random variable X be the largest of the three selected numbers. What is $P(X = 17)$? What is $P(X \geq 17)$?

We assume that each of the $\binom{20}{3}$ selections are equally likely.

The event $(X \leq x)$ occurs exactly when the three selected balls have numbers less than or equal to x . There are $\binom{x}{3}$ ways to select three such balls. Thus,

$$F(x) = P(X \leq x) = \frac{\binom{x}{3}}{\binom{20}{3}}.$$

We then compute the desired probabilities:

$$P(X = 17) = F(17) - F(16) = \frac{\binom{17}{3}}{\binom{20}{3}} - \frac{\binom{16}{3}}{\binom{20}{3}} = \frac{2}{19} \approx 0.105$$

$$P(X \geq 17) = 1 - F(16) = 1 - \frac{\binom{16}{3}}{\binom{20}{3}} = \frac{29}{57} \approx 0.509$$