

Math 262

Section 3.2

Day 20

1. Let $U \sim \text{Unif}[0, 5]$.

(a) What are the mean and variance of U ?

(b) Let $V = 3U + 2$. What are the mean and variance of V ?

(c) What do you think is the distribution of V ? Why? *Discuss with your neighbor.*

2. Let $X \sim \text{Unif}[A, B]$. Show that the mgf of X is $M_X(t) = \begin{cases} \frac{e^{Bt} - e^{At}}{(B-A)t} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases}$. Then use properties of mgfs to verify your answer for 1(c).

3. A stick of length 1 is split at a point U that is uniformly distributed on $(0, 1)$.
- (a) What is the expected length of the leftmost piece? *Discuss your reasoning with your neighbor.*

 - (b) What is the expected length of the longest piece? *Discuss your reasoning with your neighbor.*

 - (c) What is the expected length of the piece that contains the point p , $0 \leq p \leq 1$?
4. Let X be a random variable that takes on values between 0 and c .
- (a) Explain why $E(X^2) \leq cE(X)$.

 - (b) Use part (a) to show that $\text{Var}(X) \leq c^2[\alpha(1 - \alpha)]$, where $\alpha = \frac{E(X)}{c}$.

 - (c) Establish an upper bound on $\alpha(1 - \alpha)$ and conclude that $\text{Var}(X) \leq \frac{c^2}{4}$.