

Exam 2

Math 262: Probability Theory

October 2020

Name: _____

Instructions:

1. You may refer to your own notes, materials that the professor has posted on the course web site, the textbook, and computational technology (e.g., *R*, *Mathematica*, *Wolfram Alpha*). If you have questions about the exam, you may ask the professor. **Do not consult other sources, people, web sites, etc.**
 2. The St. Olaf Honor Code applies to this exam. Remember the pledge at the end of the exam.
 3. It's acceptable to leave answers in forms such as $\binom{8}{3}$ or 0.3×8.2 .
 4. Notation is as usual: P denotes probability, X is a discrete random variable and x its value, E is expected value, Var is variance, etc.
 5. Read the questions carefully. Check your work.
 6. **The exam is due Monday, October 19 at 2:00pm (classtime).** You may bring your exam to class on paper or upload it digitally via the Exam 2 link on Moodle.
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1. (9 points) What probability distribution would be most appropriate for each of the following random variables? Give the name of the distribution and the values of any parameters necessary to specify the distribution.

(a) X is the number of 3s that appear in 6 rolls of a standard, fair die.

(b) Y is chosen from the interval $[0, 1]$ in such a way that the probability that Y is in any subinterval depends only on the length of the subinterval.

(c) Z is the number of calls received by the IT Help Desk between 9am and 11am. Assume that calls occur independently, and that past data suggests an average of 8 calls per hour.

2. (12 points) In the game *Settlers of Catan*, each turn involves rolling two standard fair dice and observing their sum. If the sum is 7, then the robber is moved.

(Please ignore other aspects of the game, such as development cards or knights, that can move the robber. Assume that the robber moves once per turn, if and only if a 7 is rolled.)

- (a) What is the probability that the robber will move at least 4 times in the first 20 turns of the game?

- (b) What is the probability that the first move of the robber will occur between the 4th and the 8th turn of the game?

- (c) Let X be the number of turns required to move the robber 7 times. What is $P(X > 35 \mid X > 25)$?

- (d) What is the probability that X is within one standard deviation of its mean?

3. (15 points) The pdf of a random variable X is $f(x) = kxe^{-x^2/4}$, for $x \geq 0$ and some constant k .

(a) What is the value of k ?

(b) What are the mean and variance of X ?

(c) Find an interval $[a, a + 1]$ such that $P(a \leq X \leq a + 1) \geq 0.4$.

(d) What is the median of the distribution of X ?

4. (12 points) Suppose that random variable X has moment-generating function $M_X(t) = (1 - 3t)^{-4}$

(a) What is $E(X)$?

(b) What is $\text{Var}(X)$?

(c) Let $Y = 2X + 3$. What is the moment-generating function for Y ?

5. (12 points) A probability distribution with parameter $a > 0$ has pdf

$$f(x; a) = \begin{cases} K(1-x)x^a & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

where K is some constant.

(a) Solve for K . (Your answer will depend on a .)

(b) How does the parameter a affect the shape of this distribution?

(c) For what values of the parameter a is $E(X) > \frac{2}{3}$?

6. (8 points) Suppose the radius, R , of a circle is chosen at random from a uniform distribution on the interval $(0, 2)$. Let A be the area of the circle. (Recall $A = \pi R^2$.)
- (a) What is the expected value of A ?

(b) What is the variance of A ?

7. (6 points)

(a) If random variable X has the *memoryless property*, then what equation does X satisfy? (That is, what equation describes the memoryless property?)

(b) Which two distributions have the memoryless property?

(c) Choose one of the distribution from part (b) and show that the equation you wrote in part (a) holds for that distribution.

8. (6 points) A store has only four widgets in stock this week. The number of customers who want to buy a widget this week is a Poisson random variable with mean 3. What is the expected number of widgets that the store will sell this week?

St. Olaf Honor Pledge: I pledge my honor that on this examination I have neither given nor received assistance not explicitly approved by the professor and that I have seen no dishonest work.

Signed: _____

I have intentionally not signed the pledge. (Check the box if appropriate.)