

## Math 262

### End-of-Semester Review Problems

Day 38

1. Let  $\phi(x) = \alpha f(x) + \beta g(x)$ . Under what conditions on the constants  $\alpha$  and  $\beta$  will  $\phi(x)$  be a pdf for all possible pdfs  $f(x)$  and  $g(x)$ ?
2. Let  $X \sim \text{Exp}(\lambda)$ ,  $0 \leq s$ , and  $0 \leq t$ . Since  $X$  is memoryless, is it true that  $(X > s + t)$  and  $(X > t)$  are independent events?
3. Let  $X_1, X_2, \dots, X_{10}$  be random variables denoting bids on an item that is for sale in an auction. The item will be sold to the highest bidder. If the bids are independent and uniformly distributed between 10 and 30, what is the expected value of the sale price?
4. Suppose  $B$  and  $C$  are iid  $\text{Unif}[0, 1]$ . Find the probability that the roots of the equation  $x^2 + Bx + C = 0$  are real.

5. Alina makes 100 flips of a fair coin, and Dennis makes 99 flips of a fair coin. What is the probability that Alina gets *more* heads than Dennis?

*Hint:* Try smaller numbers. Or simulate.

6.  $X$  and  $Y$  are iid  $\text{Unif}[0, 1]$ . What is the probability that the closest integer to  $\frac{X}{Y}$  is even?

*Hint:* What is the probability that the closest integer is 0? Or 2? Or 4? Generalize.