Warm-Up: A hat contains 3 cards, identical in form, except that both sides of one card are red, both sides of another card are blue, and the third card contains one blue and one red side. One card is randomly selected from the hat and placed on a table. If the visible side of the chosen card is red, what is the probability that the other side of that card is also red?

Events: A: red-red card, B: blue-blue card, C: red-blue card

R: visible side is red

Want: $P(A \mid R) = \frac{P(A \cap R)}{P(R)} = \frac{\overset{1}{P(R \mid A)} \cdot \overset{\frac{1}{3}}{P(A)}}{P(R)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \boxed{\frac{2}{3}}$

definition of conditional probability

Bayes' Formula

Law of Total Probability $P(R) = P(R \mid A)P(A) + P(R \mid B)P(B) + P(R \mid C)P(C)$

$= 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{3} + 0 + \frac{1}{6} = \frac{1}{2}$

5. Suppose that a patient is tested for a disease. Let A be the event that the test is positive, and let D be the event that the patient actually has the disease. Further suppose that:

$P(A \mid D) = 0.99$	(*sensitivity*: probability of a positive test if the patient has the disease)
$P(A' \mid D') = 0.99$	(*specificity*: probability of a negative test if the patient doesn't have the disease)

(a) *Rare disease*: If $P(D) = 0.01$, what is the probability that a patient who tests positive actually has the disease?

Bayes' theorem $P(D \mid A) = \frac{P(A \mid D)P(D)}{P(A)} = \frac{(0.99)(0.01)}{0.99(0.01) + 0.99(0.01)} = \frac{1}{2}$

$P(A) = P(A \mid D)P(D) + P(A \mid D')P(D')$

$= (0.99)(0.01) + (0.01)(0.99)$

Imagine testing 1000 people: 990 don't have disease, 10 have the disease

- Of the 10 w/ disease, all test positive.
- Of the 990 w/out disease, about 10 test positive

$\frac{10}{20}$ pos. tests are true positives

True positive

(b) *Common disease*: If $P(D) = 0.1$, what is the probability that a patient who tests positive actually has the disease?

$$P(D \mid A) = \frac{P(A \mid D)\,P(D)}{P(A \mid D)P(D) + P(A \mid D')\,P(D')} = \frac{(0.99)(0.1)}{(0.99)(0.1) + (0.01)(0.9)} = 0.917$$

1000 people: 900 without disease, 100 with

6. A red die and a blue die are rolled. Let A be the event that the red die rolls 2, let B be the event that the sum of the rolls is 5, and let C be the event that the sum of the rolls is 7. Are A and B independent events? How about A and C?

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{4}{36} = \frac{1}{9}$$

compare

$P(A \mid B)$ and $P(A) = \frac{1}{6}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{4}{36}} = \frac{1}{4}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	$\boxed{5}$	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

↑ sums of 2 dice

$P(A) \neq P(A \mid B)$ so A and B are <u>dependent</u> events

---

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$$

$P(A) = P(A \mid C) = \frac{1}{6}$

so A and C are <u>independent</u> events