Let X be a continuous random variable with pdf $f(x)$.

EXPECTED VALUE OF X: $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

value desities

... OF $h(X)$: $E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\,dx$

SUMS $\Downarrow$ INTEGRALS

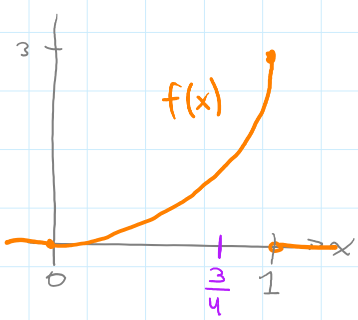
VARIANCE OF X: $\mathrm{Var}(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$= E(X^2) - E(X)^2$

MOMENT GENERATING FUNCTION: $M_X(t) = E\left(e^{tX}\right) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$

1. **Warm-up:** Let X be a random variable with pdf $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$

(a) Sketch $f(x)$. Verify that it really is a pdf.



nonegative and area = 1

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 3x^2\,dx = x^3 \Big|_0^1 = 1^3 - 0^3 = 1$$

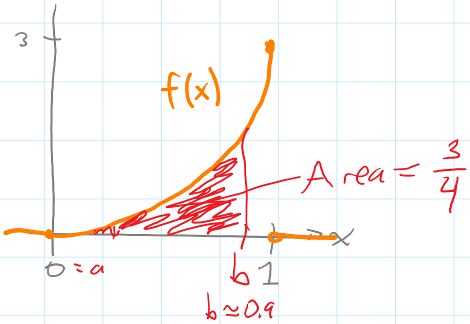
(b) Find $E(X)$ and $\mathrm{Var}(X)$.

$$E(X) = \int_0^1 x \cdot 3x^2\,dx = \int_0^1 3x^3\,dx = \frac{3}{4}x^4 \Big|_0^1 = \frac{3}{4}$$

$$E(X^2) = \int_0^1 x^2 \cdot 3x^2\,dx = \int_0^1 3x^4\,dx = \frac{3}{5}x^5 \Big|_0^1 = \frac{3}{5}$$

$$\mathrm{Var}(X) = E(X^2) - E(X)^2 = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{48-45}{80} = \frac{3}{80}$$

(c) Find an interval that contains X with probability 0.75.



Let $a=0$. Then:

WANT: $\int_0^b 3x^2\,dx = \frac{3}{4}$

$x^3\Big|_0^b = b^3 = \frac{3}{4}$

$b = \sqrt[3]{\frac{3}{4}} \approx 0.909$

Interval: $0 \le x \le \sqrt[3]{\frac{3}{4}}$