# Math 262

Section 4.5 Day 23

1. Let $X_1, X_2, \ldots, X_{300}$ be iid random variables with mean $\mu_X$ and standard deviation $\sigma_X$. Also let $T = X_1 + X_2 + \cdots + X_{300}$ and $\overline{X} = \frac{T}{300}$.

   (a) What are the values of $\mu_T$, $\sigma_T$, $\mu_{\overline{X}}$, and $\sigma_{\overline{X}}$?

   (b) What distributions are good approximations for $T$ and $\overline{X}$?

2. A farm packs tomatoes in crates. Individual tomatoes have mean weight of 10 ounces and standard deviation of 3 ounces. Estimate the probability that a crate of 40 tomatoes weighs between 380 and 410 ounces.

3. Let random variable $X$ have one of the following distributions. For what distribution of iid random variables $Y_1, Y_2, \ldots, Y_n$ is it the case that $X = Y_1 + Y_2 + \cdots + Y_n$?

   (a) $X \sim \text{Bin}(n, p)$

   (b) $X \sim \text{Gamma}(\alpha = n, \beta)$

   (c) $X \sim \text{Poisson}(\lambda = n)$

   (d) $X \sim \text{NegBin}(r = n, p)$

4. Customers at a popular restaurant are waiting to be served. Waiting times are independent and exponentially distributed with mean $1/\lambda = 10$ minutes.

   (a) What is the probability that the average wait time of the 50 customers is less than 12 minutes?

   (b) Use a normal distribution to approximate the probability that the average wait time of 50 customers is less than 12 minutes. What limit theorem justifies this?

5. Let $X_1, X_2, \ldots, X_n$ be iid random variables with an $\text{Exp}(\lambda = 2)$ distribution. Let $\mu = E(X_i)$.

   (a) What is the distribution of $T_n$? What is the value of $\mu$?

   (b) In R or *Mathematica*, write a function that computes $P\left(\left|\frac{T_n}{n} - \mu\right| \geq \epsilon\right)$ for any given parameter values $n$ and $\epsilon$.

   (c) Make a plot of $P\left(\left|\frac{T_n}{n} - \mu\right| \geq 0.01\right)$ for values of $n$ between 1 and 10,000. What limit theorem does this plot illustrate?

   (d) What is the smallest $n$ such that $P\left(\left|\frac{T_n}{n} - \mu\right| \geq 0.01\right) < 0.01$?

6. Suppose you flip a fair coin *lots* of times. What does the Law of Large Numbers say about the numbers of heads and tails you will observe?

7. Suppose that a certain casino game costs \$1 to play, and the expected winnings per game are \$0.98. What does the Law of Large Numbers say about your winnings if you play the game lots of times?

8. Fifty real numbers are each rounded to the nearest integer and then summed. If the individual round-off errors are uniformly distributed over $(-0.5, 0.5)$, then approximate the probability that the resultant sum differs from the exact sum by more than 3.

9. Suppose that a fair coin is tossed 1000 times. If the first 100 tosses all result in heads, what proportion of heads would you expect on the remaining 900 tosses? Interpret the statement "The law of large numbers swamps, but it does not compensate."