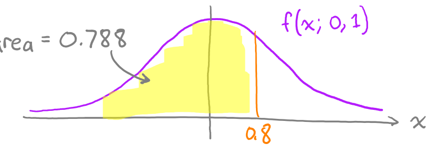
1. Let $Z$ be a standard normal random variable.

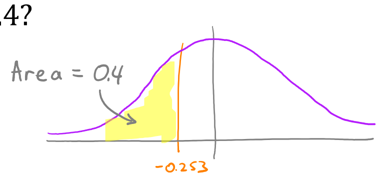
   (a) What is $P(Z \leq 0.8)$?

   R: pnorm(0.8, 0, 1) = 0.788

   or: pnorm(0.8)

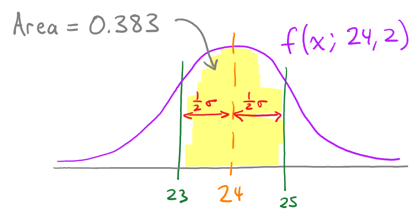
   (b) What number $c$ is such that $P(Z \leq c) = 0.4$?

   R: qnorm(0.4) = −0.253

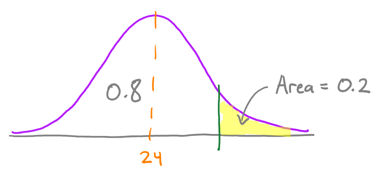
2. Let $X$ be a normal random variable with mean 24 and standard deviation 2.

   (a) What is $P(23 \leq X \leq 25)$?

   R: pnorm(25, 24, 2) − pnorm(23, 24, 2) = 0.383

   (b) What number $c$ is such that $P(X \geq c) = 0.2$?

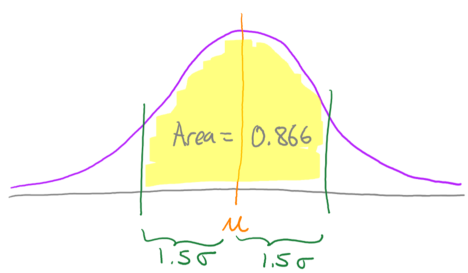
   R: qnorm(0.8, 24, 2) = 25.68 = c

   1 − 0.2

3. What is the probability that a normal random variable is within 1.5 standard deviations of its mean?

   $$\underbrace{P(\mu - 1.5\sigma < X < \mu + 1.5\sigma)}_{N(\mu,\sigma)} = \underbrace{P(-1.5 < Z < 1.5)}_{N(0,1)} = 0.866$$

   R: pnorm(1.5) − pnorm(−1.5)

4. Suppose that a fair, 6-sided die is rolled 1000 times. Use a normal distribution to approximate the probability that the number 6 appears between 150 and 200 times (inclusive).

   Let $X \sim Bin(1000, \frac{1}{6})$ be the number of 6s rolled.

   Then $E(X) = \frac{1000}{6}$ and $\sigma(X) = \sqrt{\frac{5000}{36}} \approx 11.785$.

Then $X$ is approximately $Z \sim N\left(\frac{1000}{6}, 11.8\right)$.

$\underbrace{P(150 \le X \le 200)}_{\text{Binomial distribution gives } 0.92645} \approx P(150 \le Z \le 200) = 0.919$

[Normal approximation to the binomial distribution is "good" when $np \ge 10$ and $n(1-p) \ge 10$]

**CONTINUITY CORRECTION:** $P(149.5 \le Z \le 200.5) = 0.925$

5. Let $f(x)$ denote the standard normal pdf. Estimate $f(1)$ using only the information in Table A.3 in the text.

Table A.3 gives $\Phi(z) = \int_{-\infty}^{z} f(x)\,dx$. That is, $f(x) = \Phi'(x)$.

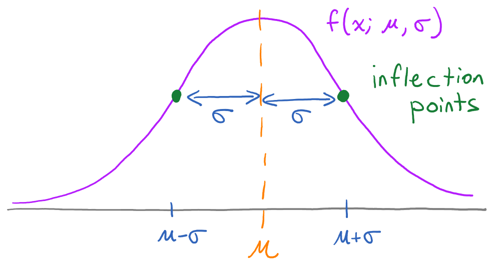
Thus:

$$f(1) \approx \frac{1}{2}\left(\frac{\Phi(1.01) - \Phi(1)}{0.01} + \frac{\Phi(1) - \Phi(0.99)}{0.01}\right) = \frac{\Phi(1.01) - \Phi(0.99)}{0.02} = \frac{0.8438 - 0.8389}{0.02}$$

$$= \frac{0.0049}{0.02} = 0.245$$

Compare with the value given by R: dnorm(1) = 0.24197

6. Let $f(x)$ denote the pdf of the $N(\mu, \sigma)$ distribution. Show that the points of inflection lie at $x = \mu \pm \sigma$. (*Hint*: differentiate twice with respect to $x$.)




$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$f'(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \frac{-(x-\mu)}{\sigma^2} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$f''(x) = \frac{-1}{\sigma^3\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} + \frac{(x-\mu)^2}{\sigma^5\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} = \frac{(x-\mu)^2 - \sigma^2}{\sigma^5\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$f''(x) = 0 \Rightarrow \frac{1}{\sigma^3\sqrt{2\pi}} = \frac{(x-\mu)^2}{\sigma^5\sqrt{2\pi}} \Rightarrow \sigma^2 = (x-\mu)^2 \Rightarrow x - \mu = \pm\sigma \Rightarrow x = \mu \pm \sigma$$

Furthermore:

| | | $\mu-\sigma$ | | $\mu+\sigma$ | | |
|---|---|---|---|---|---|---|
| $f''$ | + | | − | | + | $x$ |
| $f$ | concave up | | concave down | | concave up | |