

Math 262

More counting

Day 4

1. How many ways can you place 9 (identical) balls in 4 different boxes?
2. How many different dominoes can be formed with the numbers $1, 2, \dots, 6$? How about if the numbers $1, 2, \dots, 12$ are used?
3. How many ways can 7 identical jobs be assigned to 10 (distinct) people...
 - (a) ...if no person can do multiple jobs?
 - (b) ...if a single person can do multiple jobs?
4. Seven awards are to be distributed to 10 (distinguishable!) mathletes. How many different distributions are possible if:
 - (a) The awards are identical and nobody gets more than one?
 - (b) The awards are different and nobody gets more than one?
 - (c) Awards are identical and anyone can get any number of awards?

5. Consider the 20 “integer lattice points” (a, b) in the xy -plane given by $0 \leq a \leq 4$ and $0 \leq b \leq 3$, with a and b integers. (Draw a little picture.) Suppose you want to walk along the lattice points from $(0, 0)$ to $(4, 3)$, and the only legal steps are one unit to the *right* or one unit *up*.
- (a) How many legal paths are there from $(0, 0)$ to $(4, 3)$?
- (b) How many legal paths from $(0, 0)$ to $(4, 3)$ go through the point $(2, 2)$?
6. An box contains 5 red, 6 white, and 7 blue balls. The box is stirred and five balls are chosen without replacement. What is the probability that the 5 balls chosen include at least one of each color? Do this in steps:
- (a) Let E_1 be the event that *no red ball* is chosen, E_2 the event that *no white ball* is chosen, and E_3 the event that *no blue ball* is chosen. Find the probabilities $P(E_1)$, $P(E_2)$, and $P(E_3)$.
- (b) Find the probabilities $P(E_1 \cap E_2)$, $P(E_1 \cap E_3)$, $P(E_2 \cap E_3)$, and $P(E_1 \cap E_2 \cap E_3)$.
- (c) Use the inclusion-exclusion principle to find $P(E_1 \cup E_2 \cup E_3)$.
- (d) Use the preceding result to answer the original question.

★ **BONUS:** These are fun, but a bit more complicated than the previous problems.

- (a) How many ways can 24 students be divided into 4 groups of equal size?
- (b) What is the probability that a randomly chosen arrangement of the letters in *MISSISSIPPI* contains 4 consecutive *Is*?