

4. A factory uses 3 machines to produce certain items. Machine A produces 50% of the items, 6% of which are defective. Machine B produces 30% of the items, 4% of which are defective. Machine C produces 20% of the items, 3% of which are defective.

(a) What is the probability that a randomly-selected item is defective?

(b) If an item is defective, what is the probability that it was produced by Machine A?

5. Suppose that a patient is tested for a disease. Let A be the event that the test is positive, and let D be the event that the patient actually has the disease. Further suppose that:

$P(A | D) = 0.99$ (*sensitivity*: probability of a positive test if the patient has the disease)

$P(A' | D') = 0.99$ (*specificity*: probability of a negative test if the patient doesn't have the disease)

(a) *Rare Disease*: If $P(D) = 0.01$, what is the probability that a patient who tests positive actually has the disease?

(b) *Common Disease*: If $P(D) = 0.1$, what is the probability that a patient who tests positive actually has the disease?

★ **BONUS**: Box 1 contains 5 red balls and box 2 contains 5 blue balls. Balls are randomly removed in the following manner: after each removal from box 1, a ball is taken from box 2 (if box 2 has any balls) and placed in box 1. This process continues until all balls have been removed (so ten removals total). What is the probability that the final ball removed from box 1 is red?