

**Warm-up:** A binomial experiment is characterized by what four properties?

1. Experiment consists of  $n$  trials ( $n$  is fixed).
2. Each trial results in either "success" or "failure".
3. Trials are independent.
4. Probability of success of each trial is  $p$ .

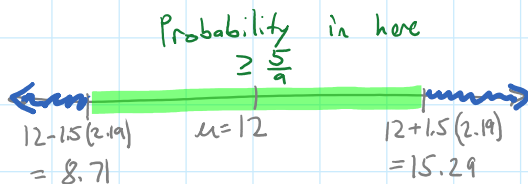
**Problem:** Let  $X \sim \text{Bin}(20, 0.6)$ . What is the probability that  $X$  is within 1.5 standard deviations of its mean?

**Chebyshev's Inequality:**  $P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$

$$\mu = np = 20(0.6) = 12$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{20(0.6)(0.4)} = 2.19$$

$$k = 1.5 \quad \text{Then: } P(|X - 12| \geq 1.5(2.19)) \leq \frac{1}{(1.5)^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$



$$P(|X - 12| \leq 1.5(2.19)) \geq \frac{5}{9}$$

Since we know  $X \sim \text{Bin}(20, 0.6)$ , we can give a more precise answer:

$$\begin{aligned}
 P(8.71 \leq X \leq 15.29) &= P(X \text{ is } 9, 10, 11, 12, 13, 14, \text{ or } 15) \\
 &= \sum_{k=9}^{15} \binom{20}{k} (0.6)^k (0.4)^{20-k} = 0.8925
 \end{aligned}$$