

1. The video for today presented a discrete-time queue simulation. At time 0, the queue contains one individual. In each time interval,  $X$  individuals enter the queue and  $Y$  individuals exit the queue, where both  $X$  and  $Y$  are Poisson random variables with mean 5.

(a) Modify the code so that the queue has a maximum size of 100. That is, if 100 individuals are in the queue, no more may join until some leave.

R:

```

1 # queue simulation as a function,
2 # with max queue size 100,
3 # stops when queue is empty
4 queueSimulation <- function(){
5   queueSize <- 1
6   time <- 0
7   while(queueSize > 0){
8     # individuals enter the queue
9     x <- rpois(1, 5)
10    if(queueSize + x > 100){
11      queueSize <- 100
12    } else {
13      queueSize <- queueSize + x
14    }
15    # individuals leave the queue
16    y <- rpois(1, 5)
17    if(y > queueSize){
18      queueSize <- 0
19    } else {
20      queueSize <- queueSize - y
21    }
22    # increment time
23    time <- time + 1
24  }
25  return(time)
26 }
27
28 # estimate time at which queue is empty
29 times = replicate(1000, queueSimulation())
30 print(times)
31 print(mean(times))

```

Mathematica:

```

In[1]:= queueSim[] := Module[{},
  queueSize = 1;
  time = 0;
  While[queueSize > 0,
    time += 1;
    x = RandomVariate[PoissonDistribution[5]];
    queueSize = queueSize + x;
    If[queueSize > 100, queueSize = 100];
    y = RandomVariate[PoissonDistribution[5]];
    If[y > queueSize, queueSize = 0, queueSize = queueSize - y];
    (*Print["at time ",time," the queue contains: ",queueSize]*)
  ];
  (*Print["time until the queue is empty: ",time];*)
  Return[time]
]

In[2]:= queueSim[]
Out[2]= 17

In[3]:= vals = Table[queueSim[], 10000]

In[4]:= Mean[vals] // N
Out[4]= 52.3655

In[5]:= StandardDeviation[vals] // N
Out[5]= 245.383

```

(b) Let  $T$  be the first time at which the queue is empty. Estimate  $E(T)$ .

Averaging over many simulations, we find  $E(T) \approx 52$ .

(c) Let  $Z$  be the first time at which the size of the queue is first reaches 20. Estimate  $E(Z)$ .

Averaging over many simulations, we find  $E(Z) \approx 54$ .

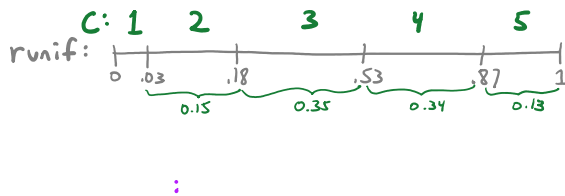
2. Suppose that  $C$ , the number of chips awarded in the game Plinko, has the following distribution:

$c$	1	2	3	4	5
$p(c)$	.03	.15	.35	.34	.13

What are two ways of simulating values of  $C$  in R?

### 1. Inverse CDF Method

Use `runif()`, then use inverse CDF to determine  $C$ .



### 2. Use `sample()`

`sample(elements, size, replace = FALSE, prob = NULL)`  
 ↑ vector of weights

```
nchips <- 1:5
pchips <- c(.03, .15, .35, .34, .13)
sample(nchips, 1, TRUE, pchips)
```

```
chips <- NULL
for(i in 1:10000){
  u <- runif(1)
  if(u < 0.03){
    x <- 1
  } else if(u < 0.18){
    x <- 2
  } else if(u < 0.53){
    x <- 3
  } else if(u < 0.87){
    x <- 4
  } else{
    x <- 5
  }
  #print(x)
  chips[i] <- x
}
hist(chips)
print(c(mean(chips), sd(chips)))
```

Use simulation to estimate the mean and standard deviation of  $C$ .

True mean: 3.39      SD: 0.978

3. Suppose that  $X$ , the winnings for one chip in Plinko, has the following distribution:

$x$	\$0	\$100	\$500	\$1000	\$10,000
$p(x)$	.39	.03	.11	.24	.23

Write a simulation of Plinko, taking into account both the number of chips a contestant earns and the amount of money won on each chip.

```
winnings <- NULL
nchips <- 1:5
pchips <- c(.03, .15, .35, .34, .13)
nwin <- c(0, 100, 500, 1000, 10000)
pwin <- c(.39, .03, .11, .24, .23)
for(i in 1:100){
  chips <- sample(nchips, 1, TRUE, pchips) #number of chips the contestant earns
  amounts <- sample(nwin, chips, TRUE, pwin) #amounts won from each chip
  winnings[i] <- sum(amounts)
}
hist(winnings)
```

What is the probability that a contestant wins more than \$11,000?

```
sum(winnings > 11000)
mean(winnings > 11000)
```

probability is about 0.3