

Math 262

Section 3.4.2

Day 23

1. Suppose that the time between goals in a hockey game is exponentially distributed with mean 18 minutes (ignore timeouts and stoppages). Let X be the time from the start of the game until the second goal occurs.

(a) Sketch the pdf of X .

(b) What is the probability that the second goal occurs less than 30 minutes after the game starts?

2. Suppose that a call center receives calls according to a Poisson distribution at a rate of 2 calls per minute. Let Y be the time between 10:00am and the 5th call received after 10:00am.

(a) Sketch the pdf of Y .

(b) What are the mean and variance of Y ?

(c) What is $P(Y < 1)$?

3. For large α , the gamma distribution converges to a normal distribution with mean $\alpha\beta$ and variance $\alpha\beta^2$. Investigate this in the case that $\beta = 1$.

(a) Let $X \sim \text{Gamma}(10, 1)$. Use technology to compute $P(X \leq x)$, for various values of x .

(b) Let $Z \sim N(10, \sqrt{10})$. Use technology to compute $P(Z \leq x)$ for the same values of x that you used in part (a). Do you find the probabilities to be close to what you found in part (a)?

(c) Now choose a larger value of α , such as $\alpha = 100$. Compute several probabilities to verify that $X \sim \text{Gamma}(\alpha, 1)$ has nearly the same distribution as $Z \sim N(\alpha, \sqrt{\alpha})$.

4. The *skewness coefficient* of the distribution of random variable X is defined

$$\gamma = \frac{E[(X - \mu)^3]}{\sigma^3}$$

Discuss at your table: How could you compute the skewness of $X \sim \text{Gamma}(\alpha, \beta)$? Then compute the skewness of X .

★ **BONUS:** Why is $\Gamma(\frac{1}{2}) = \sqrt{\pi}$? First, use the definition of $\Gamma(\alpha)$ to express $\Gamma(\frac{1}{2})$ as an integral. Then make the change of variables $y = \sqrt{2x}$ and relate the resulting expression to the normal distribution.