Math 262

Review Problems Day 24

1. Let $X \sim \text{Geom}(p)$. Find the expected value of $\frac{1}{X}$.

2. Suppose that $X \sim \text{Exp}(3)$, and let $Y = \lfloor X \rfloor$ denote the largest integer that is less than or equal to X. For example, $\lfloor 2.1 \rfloor = 2$, $\lfloor 5.99 \rfloor = 5$, and $\lfloor 14 \rfloor = 14$.

   (a) Is Y a discrete or continuous random variable?

   (b) Find $P(Y \leq 1)$.

   (c) Find $P(Y = 2)$.

   (d) Can you generalize? What is $P(Y = n)$, for any positive integer n? Is the distribution of Y one of the distributions that we have studied in this course?

3. (a) Give an example of a random variable such that $E(X)$ is undefined. (i.e., $E(X)$ diverges to ∞.)

   (b) Give an example of a random variable X such that $E(X) < \infty$ and $E(X^2)$ is undefined. (i.e., $E(X^2)$ diverges to ∞.)

4. Choose a point uniformly at random in a unit square (i.e., a square of side length 1). Let X be the distance from the point chosen to the nearest edge of the square. Find the cdf of X. (*Hint*: draw a picture!) Then find the pdf of X.