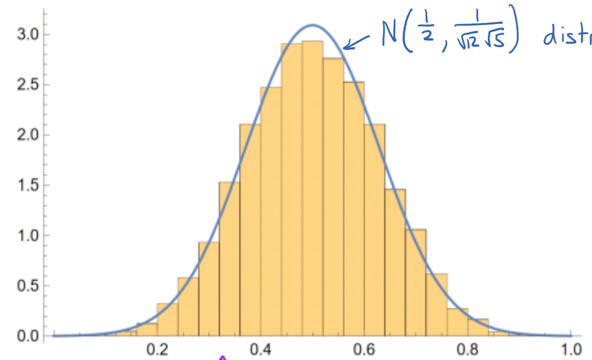
1. Simulate 10,000 averages, each of k samples from a Unif[0,1] distribution. Make a histogram of the 10,000 averages. Start with $k = 1$ and then try larger values of k. How does the shape of the histogram depend on k?

larger $k \Longrightarrow$ closer to normal

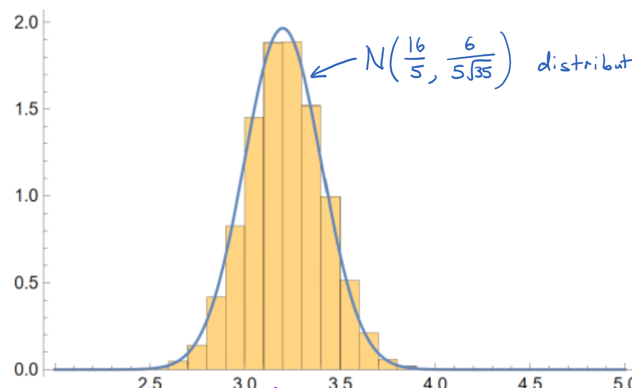


$N\left(\frac{1}{2}, \frac{1}{\sqrt{12}\sqrt{5}}\right)$ distribution

Histogram of 10,000 samples, each a sum of 5 Unif[0, 1] random variables.

2. Repeat the previous simulation, but now replace Unif[0,1] with a different distribution of your choice. What is the shape of the histogram? How does it depend on k?

again,
larger $k \Longrightarrow$ closer to normal



$N\left(\frac{16}{5}, \frac{6}{5\sqrt{35}}\right)$ distribution

Histogram of 10,000 samples, each a sum of 40 Hypergeometric$(n=8,\ M=20,\ N=50)$ random variables.

3. Let $X_1, X_2, \ldots, X_{300}$ be iid random variables with mean μ_X and standard deviation σ_X. Also let $T = X_1 + X_2 + \cdots + X_{300}$ and $\bar{X} = \frac{T}{300}$.

(a) What are μ_T, σ_T, $\mu_{\bar{X}}$, and $\sigma_{\bar{X}}$?

$\mu_T = 300\mu_X$ $\qquad$ $\sigma_T = \sigma_X\sqrt{300}$

$\mu_{\bar{X}} = \mu_X$ $\qquad$ $\sigma_{\bar{X}} = \dfrac{\sigma_X}{\sqrt{300}}$

(b) What distributions are good approximations for T and $\bar{X}$?

T is approx. $N(300\mu_X, \sigma_X\sqrt{300})$, $\bar{X}$ is approx $N\left(\mu_X, \frac{\sigma_X}{\sqrt{300}}\right)$

4. Use the Convolve function in Mathematica to plot the pdf of $X_1 + X_2 + \cdots + X_n$, where each $X_i \sim Unif[0,1]$ and $n \in \{1, 2, 3, 4, 5, 6\}$. Compare each pdf with the pdf of a normal distribution.

See Mathematica notebook.

5. A farm packs tomatoes in crates. Individual tomatoes have mean weight of 10 ounces and standard deviation of 3 ounces. Estimate the probability that a crate of 40 tomatoes weighs between 380 and 410 ounces.

T_{40} is approximately $N(400, 18.97)$ $3\sqrt{40}$

$P(380 < T_{40} < 410) \approx 0.555$

R: pnorm(410, 400, 18.97) − pnorm(380, 400, 18.97)