Convergence of Fourier Series

Math 330
5 Oct. 2017

(worksheet from last time, continued)

2. For piecewise smooth $f(x)$, the Fourier series of $f(x)$ is continuous and converges pointwise to $f(x)$ for $-L \le x \le L$ if and only if $f(-L) = f(L)$ and $f(x)$ is continuous.

3. For piecewise smooth $f(x)$, the Fourier cosine series of $f(x)$ is continuous and converges pointwise to $f(x)$ for $0 \le x \le L$ if and only if $f(x)$ is continuos.

4. For piecewise smooth $f(x)$, the Fourier sine series of $f(x)$ is continuous and converges pointwise to $f(x)$ for $0 \le x \le L$ if and only if $f(0) = 0$, $f(L) = 0$, and $f(x)$ is continuous.

5. $f(x) = e^x$

Fourier cosine series on $[0, L]$: $\quad e^x \sim \dfrac{e^L - 1}{L} + \displaystyle\sum_{n=1}^{\infty} \frac{2L}{L^2 + n^2\pi^2}\left(e^L(-1)^n - 1\right)\cos\left(\frac{n\pi x}{L}\right)$

Fourier sine series on $(0, 1)$: $\quad e^x \sim \displaystyle\sum_{n=1}^{\infty} \frac{2n\pi}{L^2 + n^2\pi^2}\left(1 - e^L(-1)^n\right)\sin\left(\frac{n\pi x}{L}\right)$

Differentiating the cosine series term-by-term produces the sine series.

Differentiating the sine series term-by-term does <u>not</u> produce the cosine series — why not?