Math 330 — 5 September 2019

ODE REVIEW WORKSHEET

1. **Linear ODE:** $a_n(t)\,y^{(n)}(t) + \cdots + a_1(t)\,y'(t) + a_0(t)\,y(t) = F(t)$

Order: highest derivative in the ODE

Constant Coefficients: if $a_0(t), \ldots, a_n(t)$ are constants

Homogeneous: if $F(t) = 0$, otherwise the ODE is nonhomogeneous

2. $y'' + y' - 6y = 0 \Rightarrow m^2 + m - 6 = 0 \Rightarrow$ solutions: $y_1(t) = e^{-3t}$

$(m+3)(m-2) = 0$ $\qquad y_2(t) = e^{2t}$

3. $y'' + 2y' + y = 0 \Rightarrow m^2 + 2m + 1 = 0 \Rightarrow$ solutions: $y_1(t) = e^{-t}$

$(m+1)^2 = 0$ $\qquad y_2(t) = te^{-t}$

(a) general solution: $y(t) = c_1 e^{-t} + c_2 t e^{-t}$

(b) IVP: $y(0) = 1 \Rightarrow 1 = c_1$ so $y(t) = e^{-t} + c_2 t e^{-t}$

$y'(t) = -e^{-t} + c_2\left(e^{-t} - te^{-t}\right)$

$y'(0) = -1 \Rightarrow -1 = -1 + c_2$

$0 = c_2$

Thus, $\boxed{y(t) = e^{-t}.}$

4. $y'' - 4y' + 5y = 0$

(a) $m^2 - 4m + 5 = 0 \Rightarrow m = \dfrac{4 \pm \sqrt{16 - 4(5)}}{2} = 2 \pm i \Rightarrow y(t) = c_1 e^{2t}\sin(t) + c_2 e^{2t}\cos(t)$

(b) BVP: $y(0) = 0 \Rightarrow 0 = c_1(0) + c_2(1) \Rightarrow c_2 = 0$ so $\boxed{y(t) = c_1 e^{2t}\sin(t)}$

$y(2) = 1 \Rightarrow 1 = c_1 e^4 \sin(2) \Rightarrow c_1 = \dfrac{1}{e^4 \sin(2)} \approx 0.0201\ldots$

5. $y' + 2ty = t$

Integrating factor method:

$\mu(t) = e^{\int 2t\,dt} = e^{t^2}$

$\frac{dy}{dt} e^{t^2} + 2t e^{t^2} y = te^{t^2}$

product rule

$\frac{d}{dt}\left(ye^{t^2}\right) = te^{t^2}$

$ye^{t^2} = \int te^{t^2}\,dt = \frac{1}{2}e^{t^2} + C$

$\boxed{y(t) = \frac{1}{2} + Ce^{-t^2}}$

Prof. Wright's video on the Integrating Factor Method: https://youtu.be/64QMx1aigtU

6. $y(t) = c_1 e^{-t} + c_2 e^{2t} + c_3 t e^{2t}$

$$(m+1)\,(m-2)\,(m-2) = (m+1)(m^2-4m+4) = m^3 - 3m^2 + 4$$

Thus: $\boxed{y''' - 3y'' + 4y = 0}$