Exam 2 Guidance

MATH 126 A/B • Fall 2025

The following is intended to help you focus your studying for Exam 2, which is on all the material we've studied through Friday, October 24. The following notes might not encompass everything you could see on the exam, but they are intended to help you think about the course content and study effectively.

Important Notes

- You may use one side of a 4×6-inch (10×15 cm) card (or a piece of paper of that side) containing notes that you prepare in advance. Anythink you want can go on that card.
- Calculators will not be helpful and definitely *not necessary.* This exam will test calculus concepts, not your ability to do arithmetic. However, you may use a calculator, but only simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone or computer.
- As you study, focus on fluency and deep understanding of calculus concepts rather than only pure memorization.

What should I know and be able to do?

"Basics" and prerequisite knowledge

- Understand the basics of taking limits, since these come up when doing improper integrals, the integral test, the divergence test, the limit comparison tests, and the ratio test. In particular as n increases, logarithms are less than polynomials, polynomials are less than exponentials, and exponentials areless than factorials. Within the realm of powers, higher powers win. Within the realm of exponentials, higher bases win. If you are taking the limit of a fraction, and the numerator and denominator "balance," then you only use the coefficients of the dominant terms.
- Be able to graph and/or identify the graphs of several basic functions: all lines $y = mx + b$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = \ln(x)$, $y = \cos(x)$, $y = \sin(x)$.
- Be able to find derivatives and antiderivatives, including using integration methods from the last exam (substitution and integration by parts).

Improper Integrals

- Remember to *check all definite integrals* to determine whether or not they are improper. Then, if so, be able express the integral as a limit. If necessary, you may need to write out a sum of several improper integrals.
- Be able to determine if an improper integral converges or diverges by first calculating an antiderivative, then using the FTC, then finally finding a limit.

- **You may use** the fact that you know the values of p for which $\int_1^\infty \frac{1}{x^p}\,dx$ converges or diverges.

Sequences and Series "Basics"

- Understand what an infinite sequence is and know what it means for a sequence to converge/diverge.
- You should (often) be able to find a closed formula for the individual terms of a sequence by detecting a pattern.
- Understand what an infinite series is and how they are different from sequences (yet related).
- Understand how the sequence of partial sums $\{s_n\}$ is related to the series $\sum_{n=1}^{\infty} a_n$ and the *definition* of what it means for the series $\sum_{n=1}^{\infty} a_n$ to converge or diverge.

Series Recognition and Testing

- Be able to recognize when a series is geometric or not geometric. You should be able to find both the *partial sum* of a geometric series and (if it converges) the *overall sum* of a geometric series. You should know which values of r allow for the series to converge.
- Be able to recognize when a series is a p-series and know when these converge/diverge. A p-series is allowed to be multiplied by a *constant* and still be considered a p-series.
- Know the Divergence Test: when you can use it, how to perform it, and how to draw a conclusion from performing it. *You should know when the test is inconclusive, and not conclude that the series converges.* You should NEVER EVER EVER use the Divergence Test to make a conclusion about $\sum_{n=1}^{\infty} a_n$ if you find that $\lim_{n\to\infty} a_n = 0$.
- Know the Integral Test: when you can use it, how to perform it, and how to draw a conclusion from performing it.
- Know the Limit Comparison Test: when you can use it, how to perform it, and how to draw a conclusion from performing it. If $a_n \approx b_n$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge. Given a_n, you should be able to pick b_n so that you know whether or not $\sum_{n=1}^{\infty} b_n$ converges or diverges.
- Know the Ratio Test: when you can use it, how to perform it, and how to draw a conclusion from performing it. You should know when the test is *inconclusive*. Be aware that this test is most often used when the terms of the series contain exponentials and/or factorials.

- For any test that you use on series, here is what needs to be written:

  1. Write down the name of the test you are using.

  2. Perform the test (i.e. do an integral or a limit)

  3. While performing the test, if you claim an integral converges/diverges or a series converges/diverges, then say how you know.

  4. Finally, once the test is performed, make a conclusion (with reasoning) as to what is happening with the *posed* problem.

Power Series and Taylor Series

- Explain what a power series is, and be able to use the ratio test to determine and interval of values of x for which a power series converges. Identify the center and radius of convergence of a power series.

- Find power series (such as Taylor series) that converge to various functions.

- Use substitution, derivatives, and antiderivatives to modify a given power series, obtaining a new power series that converges to a function.

- Find the coefficients of a Taylor/Maclaurin series for a specified function.

- Understand how a Taylor/Maclaurin series encodes the derivatives of a function at the center of the interval of convergence.

- Recognize a summation as a Taylor/Maclaurin series and use this to determine the value to which the summation converges.

- Use Taylor/Maclaurin series to approximate a function near a point.

Common mistakes to watch out for

- It may be tempting to try to combine antiderivatives with derivative rules. There are no easy "rules" for antidifferentiation, only techniques! In particular, there is no such thing as a product rule or chain rule for antiderivatives.

- Small algebra mistakes happen. We'll learn from them but move on. Be careful not to make *fundamental* algebra mistakes. Square roots and other powers can't be distributed on addition, and neither can sine, cosine, or logarithms. For example:

$$
\begin{aligned}
\sqrt{x^2+9} \quad &\text{IS NOT} \quad x+3 \\
(x+4)^3 \quad &\text{IS NOT} \quad x^3+4^3 \\
\frac{a}{b+c} \quad &\text{IS NOT} \quad \frac{a}{b}+\frac{a}{c} \\
\ln(a+b) \quad &\text{IS NOT} \quad \ln a+\ln b
\end{aligned}
$$

- People often *overuse* natural logs for antiderivative. Just because you see a fraction "$\frac{1}{\text{something}}$" does not mean natural log is even part of the answer. For example:

$$\int \frac{1}{x^2+4x+4}\,dx \quad \text{IS NOT} \quad \ln|x^2+4x+4|+C$$
$$\int \frac{1}{\text{something}}\,dx \quad \text{IS NOT} \quad \ln|\text{something}|+C$$

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section of *Active Calculus* again and think about whether or not the main ideas make intuitive sense. Can you explain them out loud?

- The problems in each section of *Active Calculus* are great practice problems. There is also a large set of practice problems available on Edfinity.

- Do a few problems each day, ramping up as we get closer to the exam day. Talk with your classmates about the problems. Talk with me and visit the help sessions if you want to make sure things are correct or if you want to chat.

- Work on fluency! The exam is timed and you want to be able to do some of the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.

- COME VISIT PROF. WRIGHT AND ASK QUESTIONS.