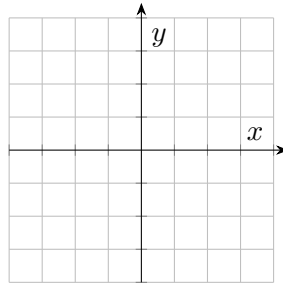


Dot Product

1. (a) In $\mathbb{R}^2$, draw the line L through the points $A = (0, 0)$ and $B = (3, 2)$. What is the slope of L ?



- (b) Draw the line M through point A that is *perpendicular* to L . What is the slope of line M ?

- (c) Find a vector $\mathbb{R}^2$ that is *parallel* to L . How is your vector related to the slope of L ?

🔖 Maybe use A and B ?

- (d) Now, try to find a vector that is *parallel* to the line M (and thus perpendicular to L). How is this vector related to the previous vector that you found?

2. (a) What is $\langle 3, 1 \rangle \cdot \langle 1, 2 \rangle$?

- (b) What is $\langle -2, 1, 0 \rangle \cdot \langle 1, 0, 1 \rangle$?

- (c) What is $\langle 3, 2 \rangle \cdot \langle 2, -3 \rangle$?

🔖 Have you seen this recently?

- (d) What is $\langle a, b, c \rangle \cdot \langle a, b, c \rangle$? How is this related to $|\langle a, b, c \rangle|$?

🔖 Remember, $|\langle a, b, c \rangle|$ is the length of $\langle a, b, c \rangle$.

3. **Group chat:** Make a conjecture by filling in the blank.

Vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular (or *orthogonal*) exactly when $\mathbf{u} \cdot \mathbf{v}$ equals _____.

4. **Group experiment:** If $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors, how does $\mathbf{u} \cdot \mathbf{v}$ relate to $|\mathbf{u}|$ and $|\mathbf{v}|$?

🔖 Choose some parallel vectors and see what happens!

5. Group chat: Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors of the same dimension, and c is a number. Which of the following statements are (always) true? Which statements are false?

(a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

(b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

(c) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

(d) $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{v} \cdot \mathbf{u})$

6. Let $\mathbf{u} = \langle 2, 0, 4 \rangle$ and $\mathbf{v} = \langle -1, 2, 3 \rangle$. If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos \theta$.

7. Find the angle between the vectors $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$.

8. Find the angle between the vectors $\langle 1, 0, 1 \rangle$ and $\mathbf{i}$.