Cross Product

1. Compute the following cross products

(a) $\langle 1, 0, 0\rangle \times \langle 0, 1, 0\rangle$

(b) $\langle a, 0, 0\rangle \times \langle 0, b, 0\rangle$

(c) $\langle 1, 1, 1\rangle \times \langle 2, 3, 4\rangle$

2. If you did #1(c) correctly, you should have gotten $\langle 1, -2, 1\rangle$.

Now compute both $\langle 1, 1, 1\rangle \cdot \langle 1, -2, 1\rangle$ *and* $\langle 2, 3, 4\rangle \cdot \langle 1, -2, 1\rangle$. What do you observe?

3. Group chat: Examine all of the cross products you computed in #1. Make a conjecture about how $\mathbf{a} \times \mathbf{b}$ is related to vectors $\mathbf{a}$ and $\mathbf{b}$.

4. What is $\langle 2, 3, 4\rangle \times \langle 1, 1, 1\rangle$?

5. Group chat: What do you think is the relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$?

6. Experiment time: If $\mathbf{a}$ and $\mathbf{b}$ are parallel vectors, what can you say about $\mathbf{b} \times \mathbf{a}$?

7. Consider the points $A = (5,2,0)$, $B = (2,6,1)$, $C = (2,4,7)$, and $D = (5,0,6)$.

(a) Sketch these points in $\mathbb{R}^3$ and show that they form a parallelogram.

(b) Use a cross product to find the area of this parallelogram.

8. Find the area of the parallelogram with vertices $(0,0)$, $(5,-2)$, $(7,1)$, and $(2,3)$.

These points are in $\mathbb{R}^2$. Is that a problem?

9. **Cleo:** Hey Bastian, remember how a line is determined by two points?

Bastian: I sure do! Did you know that in $\mathbb{R}^3$ a plane is determined by three points?

Cleo: Yes—as long as all three points are not all on the same line.

Group chat: Does this make sense?

Challenge question: The plane P passes through the points $A = (-1,2,3)$, $B = (0,3,4)$, and $C = (1,5,7)$. Can you find a vector $\mathbf{n}$ that is orthogonal (perpendicular) to the plane P?