MATH 126 — 12 November 2025

EXAMPLE: Wind chill $w(T, v)$ is determined by the temperature T and wind speed v.

(see the chart on the next page)

Recall from Calculus I:

If $y = f(x)$, the derivative $f'(x)$ or $\frac{dy}{dx}$ measures the rate at which y changes relative to x.

limit definition: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Now in Calculus II:

$z = f(x,y)$, a function with two (or more!) inputs.

If f has several inputs, you might want to know the rate at which f changes relative to <u>one</u> of the inputs (the other input stays constant).

PARTIAL DERIVATIVES:

These are derivatives just like in Calc I, but now you differentiate with respect to one variable and pretend that the other variables are constant.

With respect to x: $f_x = \dfrac{\partial f}{\partial x} = \lim\limits_{h \to 0} \dfrac{f(x+h, y) - f(x,y)}{h}$

with respect to y: $f_y = \dfrac{\partial f}{\partial y} = \lim\limits_{h \to 0} \dfrac{f(x, y+h) - f(x,y)}{h}$

WIND CHILL FUNCTION $w(T,v)$

wind speed v (miles per hour)	temperature T (degrees Fahrenheit)										
	−30	**−25**	**−20**	**−15**	**−10**	**−5**	**0**	**5**	**10**	**15**	**20**
5	−46	−40	−34	−28	−22	−16	−11	−5	1	7	13
10	−53	−47	−41	−35	−28	−22	−16	−10	−4	3	9
15	−58	−51	−45	−39	−32	−26	−19	−13	−7	0	6
20	−61	−55	−48	−42	−35	−29	−22	−15	−9	−2	4
25	−64	−58	−51	−44	−37	−31	−24	−17	−11	−4	3
30	−67	−60	−53	−46	−39	−33	−26	−19	−12	−5	1
35	−69	−62	−55	−48	−41	−34	−27	−21	−14	−7	0
40	−71	−64	−57	−50	−43	−36	−29	−22	−15	−8	−1

Partial Derivatives

1. Look up at the *wind chill function* $w(T, v)$ on the screen. What is $w(-20, 15)$? What does this mean?

Remember that T is temperature and v is wind speed.

$w(-20, 15) = -45$

If the actual temperature is -20 and the wind speed is 15, then it feels like the temperature is -45.

2. For all of the questions below, suppose that $T = 10$ and $v = 15$. Use the wind chill function on the screen to answer the following:

(a) If $\Delta T = 5$, what is Δw?

$$\Delta w = w(15, 15) - w(10, 15) = 0 - (-7) = 7$$

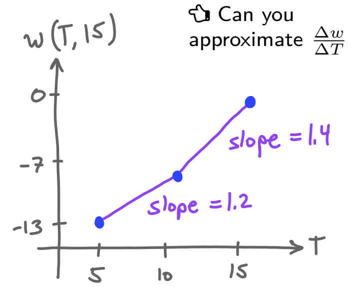
(b) If $\Delta T = -5$, what is Δw?

$$\Delta w = w(5, 15) - w(10, 15) = -13 - (-7) = -6$$

(c) **Group chat:** What is your best guess for Δw if $\Delta T = 1$? Why?

Can you approximate $\frac{\Delta w}{\Delta T}$?

$$\frac{\Delta w}{\Delta T} \approx \frac{7}{5} = 1.4$$



(d) **Group chat:** What is your best guess for Δw if $\Delta T = -1$? Why?

$$\frac{\Delta w}{\Delta T} \approx \frac{-6}{-5} = 1.2$$

(e) What is your best guess for the derivative of w in the direction of T?

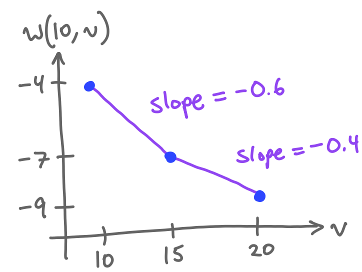
Average the previous two answers: $\frac{1.4 + 1.2}{2} = 1.3$

(f) If $\Delta v = 5$, what is Δw?

$$\Delta w = w(10, 20) - w(10, 15) = -9 - (-7) = -2$$

(g) If $\Delta v = -5$, what is Δw?

$$\Delta w = w(10, 10) - w(10, 15) = -4 - (-7) = 3$$



(h) **Group chat:** What is your best guess for Δw if $\Delta v = 1$? Why?

$$\frac{\Delta w}{\Delta v} \approx \frac{-2}{5} = -0.4$$

(i) **Group chat:** What is your best guess for Δw if $\Delta v = -1$? Why?

$$\frac{\Delta w}{\Delta v} \approx \frac{3}{-5} = -0.6$$

(j) What is your best guess for the derivative of w in the direction of v?

Average the previous two answers: $\frac{-0.4 - 0.6}{2} = -0.5$

3. Review: Find the derivative of $f(x) = 2x^2 + 14$ with respect to x.

$$f'(x) = \frac{df}{dx} = 4x$$

4. Find the derivative of $f(x,y) = 2x^2 + \underbrace{14y}_{\text{constant with respect to } x}$ with respect to x, pretending that y is a constant.

Partial Derivative: $f_x = \dfrac{\partial f}{\partial x} = 4x$

5. Now pretend that x is a constant and find the derivative of $f(x,y) = \underbrace{2x^2}_{\text{constant with respect to } y} + 14y$ with respect to y.

Partial Derivative: $f_y = \dfrac{\partial f}{\partial y} = 14$

6. Suppose that $f(x,y) = x^2 + y^2 + xy^2$.

(a) What is the derivative of f with respect to x?

$$f_x = \frac{\partial f}{\partial x} = 2x + y^2$$

(b) What is the derivative of f with respect to y?

$$f_y = \frac{\partial f}{\partial y} = 2y + 2xy$$

(c) What is the derivative of f with respect to x at the point $(1,2)$?

$$f_x(1,2) = 2(1) + (2)^2 = 6$$

(d) **Lana:** OK, you that $f_x(1,2)$ is a number, but how can I *interpret* this? What does it mean?

Group chat: How would you answer Lana's question? Can you give an interpretation in words? Can you also draw a diagram to illustrate this derivative?

The rate of change of f with respect to x when $x=1$ and $y=2$ is 6.

At $(1,2)$, the slope of the graph of f in the direction of x is 6.

(e) What is the derivative of f with respect to y at the point $(1,2)$? What does this mean in words? Can you also give a graphical interpretation of this derivative?

$$f_y(1,2) = 2(2) + 2(1)(2) = 8$$

The rate of change of f with respect to y when $x=1$ and $y=2$ is 8

At $(1,2)$, the slope of the graph of f in the y-direction is 8.

We didn't get to these in class, but they are good examples/practice!

7. The *formula* for the windchill function is $w = 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}$. Use this to calculate the *exact* values of $\dfrac{\partial w}{\partial T}$ and $\dfrac{\partial w}{\partial v}$ when $(T,v) = (10,15)$.

Compare to your estimate from earlier!

$$\frac{\partial w}{\partial T} = 0.6215 + 0.4275\, v^{0.16} \qquad \frac{\partial w}{\partial T}(10,15) = 0.6215 + 0.4275(15)^{0.16} \approx 1.28$$

$$\frac{\partial w}{\partial v} = (0.4275T - 35.75)(0.16\, v^{-0.84}) \qquad \frac{\partial w}{\partial v}(10,15) = (-31.475)(0.16)(15)^{-0.84} \approx -0.518$$

close to what we estimated!

8. Let $f(x,y) = e^{x^2+y}$. Find the partial derivatives of f at the point $(2,-1)$.

$$f_x = (e^{x^2+y})(2x) \qquad f_x(2,-1) = 4e^3$$

$$f_y = (e^{x^2+y})(1) \qquad f_y(2,-1) = e^3$$

9. Find the partial derivatives of $f(x,y) = \cos(x^2 - 3y)$ at the point $\left(0, \frac{\pi}{2}\right)$.

$$f_x = -\sin(x^2 - 3y)(2x) \qquad f_x\left(0,\tfrac{\pi}{2}\right) = -\sin\left(-\tfrac{3\pi}{2}\right)(0) = 0$$

$$f_y = -\sin(x^2 - 3y)(-3) \qquad f_y\left(0,\tfrac{\pi}{2}\right) = -\sin\left(-\tfrac{3\pi}{2}\right)(-3) = 3(-1) = -3$$