Partial Derivatives

1. Look up at the *wind chill function* $w(T, v)$ on the screen. What is $w(-20, 15)$? What does this mean?

Remember that T is temperature and v is wind speed.

2. For all of the questions below, suppose that $T = 10$ and $v = 15$. Use the wind chill function on the screen to answer the following:

(a) If $\Delta T = 5$, what is Δw?

(b) If $\Delta T = -5$, what is Δw?

(c) **Group chat:** What is your best guess for Δw if $\Delta T = 1$? Why?

Can you approximate $\frac{\Delta w}{\Delta T}$?

(d) **Group chat:** What is your best guess for Δw if $\Delta T = -1$? Why?

(e) What is your best guess for the derivative of w in the direction of T?

(f) If $\Delta v = 5$, what is Δw?

(g) If $\Delta v = -5$, what is Δw?

(h) **Group chat:** What is your best guess for Δw if $\Delta v = 1$? Why?

(i) **Group chat:** What is your best guess for Δw if $\Delta v = -1$? Why?

(j) What is your best guess for the derivative of w in the direction of v?

3. Review: Find the derivative of $f(x) = 2x^2 + 14$ with respect to x.

4. Find the derivative of $f(x, y) = 2x^2 + 14y$ with respect to x, pretending that y is a constant.

5. Now pretend that x is a constant and find the derivative of $f(x, y) = 2x^2 + 14y$ with respect to y.

6. Suppose that $f(x, y) = x^2 + y^2 + xy^2$.

(a) What is the derivative of f with respect to x?

(b) What is the derivative of f with respect to y?

(c) What is the derivative of f with respect to x at the point $(1, 2)$?

(d) **Lana:** OK, you that $f_x(1, 2)$ is a number, but how can I *interpret* this? What does it mean?
Group chat: How would you answer Lana's question? Can you give an interpretation in words? Can you also draw a diagram to illustrate this derivative?

(e) What is the derivative of f with respect to y at the point $(1, 2)$? What does this mean in words? Can you also give a graphical interpretation of this derivative?

7. The *formula* for the windchill function is $w = 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}$. Use this to calculate the *exact* values of $\dfrac{\partial w}{\partial T}$ and $\dfrac{\partial w}{\partial v}$ when $(T, v) = (10, 15)$.

Compare to your estimate from earlier!

8. Let $f(x, y) = e^{x^2+y}$. Find the partial derivatives of f at the point $(2, -1)$.

9. Find the partial derivatives of $f(x, y) = \cos(x^2 - 3y)$ at the point $\left(0, \frac{\pi}{2}\right)$.