Iterated Integrals

1. Goal: We want to evaluate $\iint_R (x^2 + y^2)\, dA$, which equals the volume underneath the graph of $f(x, y) = x^2 + y^2$ and above the rectangle $R = [-2, 2] \times [-2, 2]$.

R is the rectangle of points (x, y) where $-2 \le x \le 2$ and $-2 \le y \le 2$.

(a) Suppose that c is simply a constant and $f(x) = x^2 + c^2$. Write down and evaluate the integral that gives the area between $f(x)$ and the x-axis for $-2 \le x \le 2$.

(b) **Ava:** OK, this means that $\displaystyle\int_{-2}^{2} (x^2 + y^2)\, dx = \frac{16}{3} + 4y^2$.

Milo: What, you computed a definite integral. Isn't the answer supposed to be a *number*?

Group chat: How would you answer Milo's question?

(c) **Ava:** That must mean that the actual volume underneath $f(x, y) = x^2 + y^2$ and above the rectangle R is given by

$$\int_{-2}^{2} \left(\frac{16}{3} + 4y^2 \right) dy.$$

Group chat: Do you agree with Ava's statement? Explain.

(d) How can you evaluate $\iint_R (x^2 + y^2)\, dA$?

2. Now we want to compute $\iint_R \frac{xy^3}{x^2+1}\,dA$, where R is the rectangle $[0,1]\times[-3,3]$.

Let's play a fun game: split your group in half (or as close as you can get).

(a) One half of your group should compute the double integral this way: $\int_{-3}^{3}\int_{0}^{1} \frac{xy^3}{x^2+1}\,dx\,dy$

(b) The other half of your group should compute: $\int_{0}^{1}\int_{-3}^{3} \frac{xy^3}{x^2+1}\,dy\,dx$

Do you notice the difference?

(c) Which half of your group had more fun?

I guess it depends on your definition of "fun."

3. Evaluate $\iint_R x\cos(xy)\,dA$, where $R = \{(x,y) \mid 0 \le x \le \pi, 0 \le y \le 1\}$.

Does the order of integration matter?