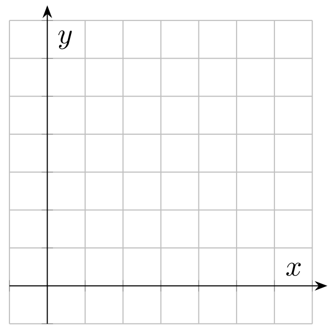
Double Integrals over General Regions

1. Our goal is to find the volume underneath $f(x,y) = x^2 + y^2$ and above the triangular region T formed by the x-axis, the line $x = 5$, and the line $y = x$.

(a) Sketch the region T:



(b) Draw a slice parallel to the y-axis through triangle T. Let x be the value at which your slice crosses the x-axis. What are the minimum and maximum values of y on your slice of triangle T?

Your answer should have an x in it.

$$\_\_\_\_\_\_\_\_\_\_ \leq y \leq \_\_\_\_\_\_\_\_\_\_$$

(c) Write an integral that gives the area below the graph of $f(x,y) = x^2 + y^2$ and above the slice you drew in part (b).

(d) What are the minimum and maximum values of x at which you could have drawn your slice in part (b)?

$$\_\_\_\_\_\_\_\_\_\_ \leq x \leq \_\_\_\_\_\_\_\_\_\_$$

(e) Now integrate your result from part (c) over the range of x-values you found in part (d). How can you interpret this new integral?

(f) Use what you found above to rewrite the following double integral as an *iterated* integral with actual bounds of integration.

$$\iint_T (x^2 + y^2)\, dA =$$

2. Evaluate the double integral $\iint_D (x + 2y)\ dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$, by following the steps below.

(a) Sketch the region D.

(b) In order to evaluate the integral, we must first write it as an iterated integral

$$\int_a^b \int_{g_1(x)}^{g_2(x)} (x + 2y)\, dy\, dx$$

Determine the x values for when the two parabolas intersect. These values are a and b.

(c) What are the inner bounds of integration $g_1(x)$ and $g_2(x)$? You can now write the double integral over D as an iterated integral.

(d) Evaluate the iterated integral by integrating the inside integral first and then the outside integral last.

3. Sketch the region whose area is given by the iterated integral

$$\int_0^4 \int_0^{\sqrt{x}} 1 \; dy \, dx.$$

Then find another iterated integral whose value is equal to this integral, but with the order of integration $dx\,dy$. Show that both integrals yield the same value.

4. Let T be the triangle with vertices $(0, 0)$, $(0, 4)$, and $(2, 4)$. Evaluate the double integral:

$$\iint_T (x + y)\; dA$$

5. Find the volume of the solid under the plane $z = x + 2y$ and above the region bounded by $y = x$ and $y = x^4$.