Math 234

Direct Proof and Counterexample — Day 7

Discuss the following problems with the people at your table.

1. Using the definition of divisibility prove this statement: For all integers a, b, and c, if $a|b$ and $a|c$ then $a|(b+c)$.

2. Consider the following statement: "The negative of any multiple of 3 is a multiple of 3."

   (a) Write the statement formally using a quantifier and a variable.

   (b) Determine whether the statement is true or false, justifying your answer.

3. For the statements below, determine whether the statement is true or false. If true, prove the statement directly using definitions. If false, provide a counterexample.

   (a) The sum of any three consecutive integers is divisible by 3.

   (b) A sufficient condition for an integer to be divisible by 8 is that it is divisible by 16.

   (c) For all integers a, b, and c, if $ab|c$ then $a|c$ and $b|c$.

   (d) For all integers a, b, and c, if $a|bc$ then $a|b$ or $a|c$.

4. The standard factored form for an integer n is $n = p_1^{e_1} p_2^{e_2} p_3^{e_3} \cdots p_k^{e_k}$ where k is a positive integer, $p_1, p_2, \ldots, p_k$ are distinct prime numbers and $e_1, e_2, \ldots, e_k$ are positive integers.

   (a) What is the standard factored form of 2022? How about 2023?

   (b) If the standard factored form for an integer a is $a = p_1^{e_1} p_2^{e_2} p_3^{e_3} \cdots p_k^{e_k}$, what is the standard factored form of a^3?

   (c) Find the least positive integer j such that $2^4 \cdot 5^5 \cdot 11 \cdot 17^2 \cdot j$ is a perfect cube.

5. How many zeros are at the end of $65^8 \cdot 56^5$? Explain how you can answer this without actually computing the number.

6. Prove that if n is a nonnegative integer whose decimal representation ends in 0, then $5|n$.