Math 234

Proof by Induction Day 10

Discuss the following problems with the people at your table.

1. Let's use mathematical induction to prove that for any integer $n \geq 0$,

$$\sum_{i=0}^{n} 2^i = 2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1.$$

   (a) Define the basis statement $P(0)$ for this proof.

   (b) Prove the basis statement $P(0)$. (This will be very short.)

   (c) Write down the statement $P(k)$, the inductive hypothesis.

   (d) Write down the statement $P(k+1)$, which is the statement we want to prove.

   (e) Finish the proof by showing that if $P(k)$ is true, then $P(k+1)$ follows.

2. Use mathematical induction to prove that for any integer $n \geq 1$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

(a) Define the basis statement $P(1)$ for this proof.

(b) Prove the basis statement $P(1)$.

(c) Write down the statement $P(k)$, the inductive hypothesis.

(d) Write down the statement $P(k+1)$, which is the statement we want to prove.

(e) Finish the proof by showing that if $P(k)$ is true, then $P(k+1)$ follows.

3. Use mathematical induction to prove that $\sum_{i=1}^{n} i \cdot 2^i = (n-1) \cdot 2^{n+1} + 2$ for any integer $n \geq 1$.

4. Use mathematical induction to prove that for any integer $n \geq 0$, $2^{2n} - 1$ is divisible by 3.

5. Use mathematical induction to prove that for any integer $n \geq 0$, $n(n^2 + 5)$ is divisible by 6.

6. **Bonus:** Find a formula in a, r, m, and n for the sum

$$ar^m + ar^{m+1} + ar^{m+2} + \cdots + ar^{m+n}.$$

Prove that your formula is correct.