Math 234

Set Theory Day 12

Discuss the following problems with the people at your table.

1. Consider the following sets: $A = \{1, 3, 6, 10\}$ and $B = \{2, 4, 6, 8\}$. Determine the following sets by writing their elements in set notation:

   (a) $A \cup B$

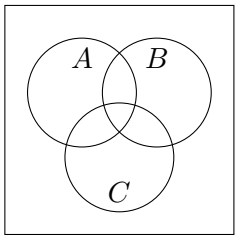
   (b) $A \cap B$

   (c) $B \cap A$

   (d) $A - B$

   (e) $B - A$

2. For each item below, copy the Venn diagram and shade the portion of the Venn diagram corresponding to the indicated set.



   (a) $A \cup B \cup C$

   (b) A^c

   (c) $A \cup B \cup C^c$

   (d) $(A \cap B) - C$

   (e) $A^c \cap B^c \cap C^c$

   (f) $(A \cup B \cup C)^c$

3. Let $A = \{x \in \mathbf{R} \mid i < x < i + 1 \text{ for some integer } i\}$.

   (a) Describe in words the set A.

   (b) Describe in words the set A^c.

4. Consider the set $A = \{n \in \mathbf{Z} \mid n \text{ is divisible by } 10\}$ and $B = \{n \in \mathbf{Z} \mid n \text{ is divisible by } 20\}$.

   (a) Prove that $B \subseteq A$.

   (b) Prove that $A \not\subseteq B$.

5. Let $C_i = \{-i, i\}$ for all nonnegative integers i.

   (a) Are C_1 and C_2 disjoint? Are $C_0, C_1, C_2, \ldots$ mutually disjoint?

   (b) $\bigcup_{i=0}^{4} C_i =?$

   (c) $\bigcap_{i=0}^{4} C_i =?$

   (d) $\bigcup_{i=0}^{n} C_i =?$

   (e) $\bigcup_{i=0}^{\infty} C_i =?$

   (f) Do the sets $C_0, C_1, C_2, \ldots$ form a partition of $\mathbf{Z}$?

6. Let $D = \{1, 4, 7\}$ and $E = \{1, 2\}$.

   (a) Write out the Cartesian product $D \times E$.

   (b) Write out the power set $\mathscr{P}(D)$.

   (c) How many elements are in $\mathscr{P}(D \times E)$?

7. If A is a set of n elements, how many elements are in $\mathscr{P}(A)$? Explain your reasoning.

8. Given any two sets C and D, describe in words the set $(C \cup D) - (C \cap D)$.

write down some examples for specific sets!

9. **Bonus:** Let $D_i = \left[0, \frac{1}{i}\right] = \left\{x \in \mathbf{R} \mid 0 \leq x \leq \frac{1}{i}\right\}$ for all positive integers i.

   (a) What is $\bigcup_{i=1}^{\infty} D_i$?

   (b) What is $\bigcap_{i=1}^{\infty} D_i$?