

Fibonacci Polynomial Identities

What did you observe?

What questions do you have?

Fibonacci Sequence

$$F_0 = 0, \quad F_1 = 1, \quad F_n = F_{n-1} + F_{n-2} \quad \text{for } n > 1$$

Today: generalize the Fibonacci Sequence

We could change: the initial values
or the recurrence.

← TODAY
← Wednesday

Lucas Sequence

$$L_0 = 2, \quad L_1 = 1, \quad \text{and} \quad L_n = L_{n-1} + L_{n-2} \quad \text{for } n > 1.$$

2, 1, 3, 4, 7, 11, 18, 29, 47, ...

Conjectures:

1. The Lucas numbers satisfy $\lim_{n \rightarrow \infty} \frac{L_{n+1}}{L_n} = \frac{1+\sqrt{5}}{2}$.
2. The Fibonacci and Lucas numbers satisfy $F_{n-1} + F_{n+1} = L_n$.
3. The Lucas numbers satisfy $\sum_{k=0}^n L_k = L_{n+2} - 1$.
4. The Fibonacci and Lucas numbers satisfy $F_n L_n = F_{2n}$.
5. The Lucas numbers satisfy $L_{2n} = L_n^2 + (-1)^{n+1} 2$.
6. The Fibonacci and Lucas numbers satisfy

is this a "polynomial" identity?

$$L_{m+1} F_n + L_m F_{n+1} = L_{m+n+1}$$

