MATH 242 — 15 October 2025

The Logistic Map

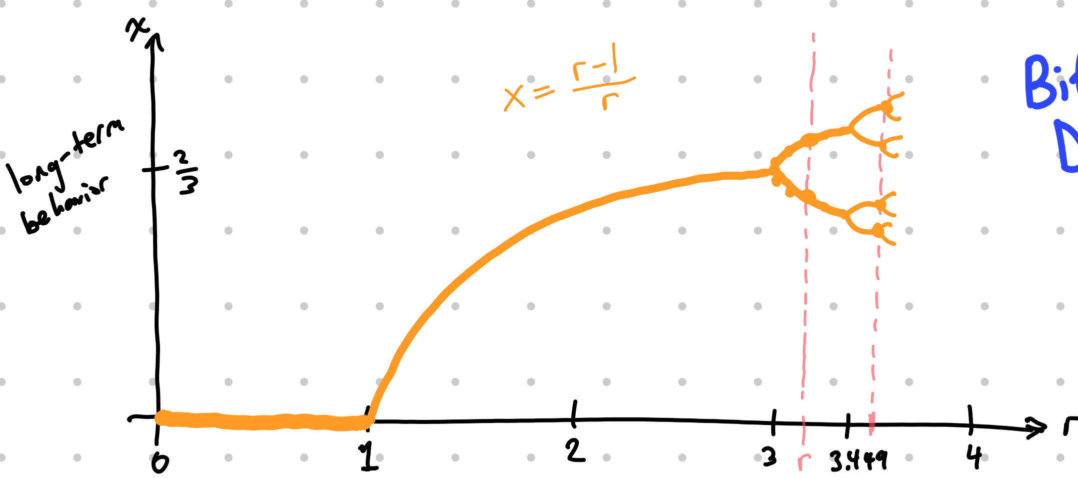
$$f_r(x) = r \cdot x(1-x)$$

$$0 \le r \le 4$$

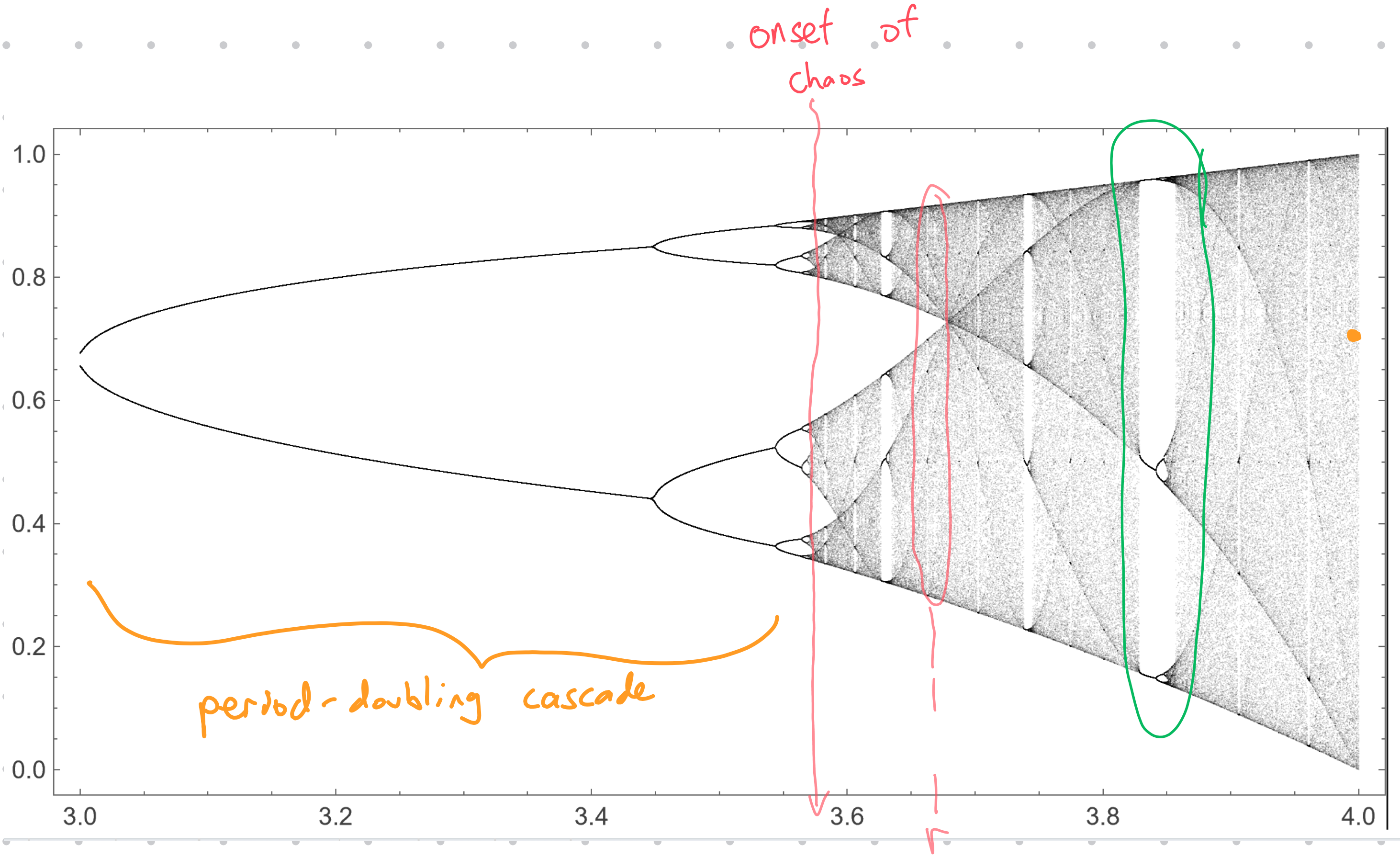
$$0 \le x \le 1$$

Recall:

- If $0 \le r \le 1$, then the trajectories converge to 0.
- If $1 < r < 3$, then the trajectories converge to $x^* = \frac{r-1}{r}$
- If $3 < r < 3.449$, then the trajectories oscillate between two values.
- If $3.449 < r < 3.544$, then the trajectories oscillate among four values.
- As r increases, we observe a sequence of period-doubling bifurcations.



Bifurcation Diagram



Chaos: For many values of $3.6 < r \leq 4$, the logistic map trajectories exhibit chaos. Mathematically, chaos has 3 properties:

1. For most starting values, the trajectory never enters a repeating cycle.
2. Trajectories visit every small interval in the domain.
3. Trajectories exhibit sensitive dependence on itital conditions.