

Finding Primes

MATH 242 Modern Computational Mathematics

Primality Testing

Write a function `isPrime(n)` that accepts a positive integer `n` and determines whether it is prime. Your function should return `True` if `n` is prime and `False` otherwise.

```
In [1]: def isPrime(n):
# first, check whether n is even
if n == 2:
    return True
if n % 2 == 0:
    return False

# check whether n is divisible by 3, 5, 7, 11, ..., n-2
for k in range(3, n//2, 2):
    if n % k == 0:
        return False

# if we get here, then the loop finished without finding a factor of
# n, so n is prime
return True
```

```
In [2]: isPrime(103)
```

```
Out[2]: True
```

Listing Primes

Use your function `isPrime` to make a list of all primes up to some number `N`.

How fast is this?

How long would it take to list all primes up to one billion?

Can you think of a faster way to list the primes?

```
In [4]: print([isPrime(n) for n in range(2,40)])
```

```
Out[4]: [True, True, False, True, False, True, False, False, False, True, False,
True, False, False, False, True, False, True, False, False, False, True,
False, False, False, False, False, True, False, True, False, False, False,
False, False, True, False, False]
```

```
In [5]: [n for n in range(2,40) if isPrime(n)]
```

```
Out[5]: [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37]
```

```
%time primeList = [n for n in range(2,10000) if isPrime(n)]  
print(primeList)
```

```
Out[7]: CPU times: user 374 ms, sys: 0 ns, total: 374 ms
```

```
Wall time: 522 ms
```

```
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71,  
73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151,  
157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233,  
239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317,  
331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419,  
421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503,  
509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607,  
613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701,  
709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811,  
821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911,  
919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997, 1009, 1013,  
1019, 1021, 1031, 1033, 1039, 1049, 1051, 1061, 1063, 1069, 1087, 1091,  
1093, 1097, 1103, 1109, 1117, 1123, 1129, 1151, 1153, 1163, 1171, 1181,  
1187, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259, 1277,  
1279, 1283, 1289, 1291, 1297, 1301, 1303, 1307, 1319, 1321, 1327, 1361,  
1367, 1373, 1381, 1399, 1409, 1423, 1427, 1429, 1433, 1439, 1447, 1451,  
1453, 1459, 1471, 1481, 1483, 1487, 1489, 1493, 1499, 1511, 1523, 1531,  
1543, 1549, 1553, 1559, 1567, 1571, 1579, 1583, 1597, 1601, 1607, 1609,  
1613, 1619, 1621, 1627, 1637, 1657, 1663, 1667, 1669, 1693, 1697, 1699,  
1709, 1721, 1723, 1733, 1741, 1747, 1753, 1759, 1777, 1783, 1787, 1789,  
1801, 1811, 1823, 1831, 1847, 1861, 1867, 1871, 1873, 1877, 1879, 1889,  
1901, 1907, 1913, 1931, 1933, 1949, 1951, 1973, 1979, 1987, 1993, 1997,  
1999, 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081, 2083,  
2087, 2089, 2099, 2111, 2113, 2129, 2131, 2137, 2141, 2143, 2153, 2161,  
2179, 2203, 2207, 2213, 2221, 2237, 2239, 2243, 2251, 2267, 2269, 2273,  
2281, 2287, 2293, 2297, 2309, 2311, 2333, 2339, 2341, 2347, 2351, 2357,  
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