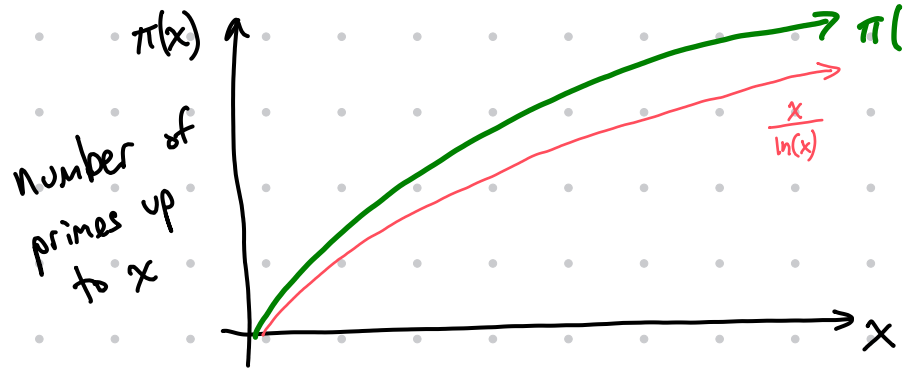
MATH 242 — 3 November 2025

How are the primes distributed?

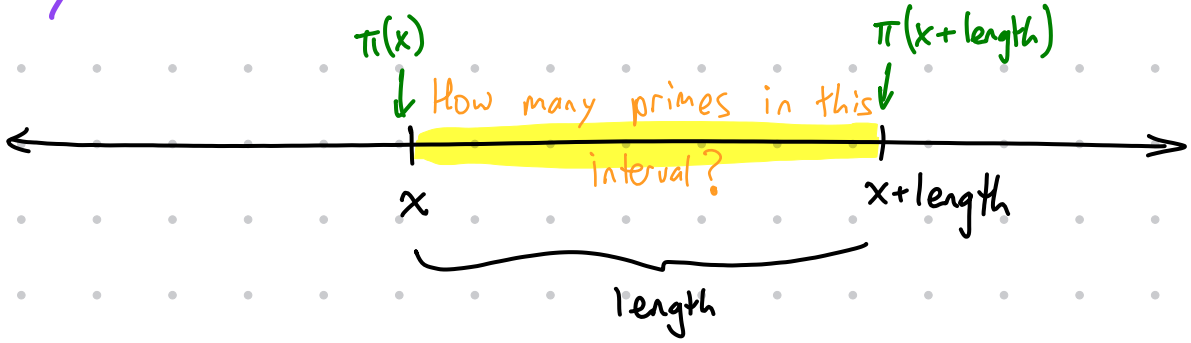
What is the shape of the prime counting function?



On Homework:

$\pi(x)$ $\quad$ $\frac{x}{\ln(x)}$

Density of Primes



$$\text{approx density of primes near } x = \frac{\pi(x+\text{length}) - \pi(x)}{\text{length}}$$

If the density of primes near x is $\frac{1}{\ln(x)}$, then the count of primes up to x is...

$$\pi(x) \approx \int_2^x \frac{1}{\ln(t)}\,dt = \text{li}(x)$$

logarithmic integral

Skewes proved: $\pi(x) > \mathrm{li}(x)$ for some x satisfying

$$x < 10^{10^{10^{34}}}$$

Recent estimates: $x \approx 1.397 \times 10^{316}$

The RIEMANN ZETA FUNCTION

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \cdots$$

Example:

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{6}$$